Calculus Chapter 6 Part 2 Review Solutions

There are 4 problems like #1, 2 on the test.

1)

$$\frac{dy}{dx} = 3x^3 + 5 \text{ and } y = 3 \text{ when } x = 0$$

$$dy = \left(3x^3 + 5\right)dx \Rightarrow \int dy = \int \left(3x^3 + 5\right)dx \Rightarrow y = \frac{3x^4}{4} + 5x + C$$

$$3 = 0 + 0 + C \Rightarrow C = 3 \qquad y = \frac{3x^4}{4} + 5x + 3$$

2) $$\frac{dy}{dx} = \frac{3 + x^3}{2y - 1} \text{ and } y = 1 \text{ when } x = 2$$

$$\left(2y - 1\right)dy = \left(3 + x^3\right)dx \Rightarrow \int \left(2y - 1\right)dy = \int \left(3 + x^3\right)dx \Rightarrow y^2 - y = 3x + \frac{x^4}{4} + C$$

$$1 - 1 = 6 + 4 + C \Rightarrow C = -10 \qquad y^2 - y = 3x + \frac{x^4}{4} - 10$$

There are 4 problems like #3 - 5 on the test.

3) $36000 = 12000e^{0.037t} \Rightarrow \ln 3 = 0.037t \Rightarrow t = \dfrac{\ln 3}{0.037} \approx 29.69$ years

4) $0.49 = 1e^{-0.045t} \Rightarrow \ln 0.49 = -0.045t \Rightarrow t = \dfrac{\ln 0.49}{-0.045} \approx 15.85$ days

5) A certain radioactive isotope has a half-life of approximately 230 years. How many years, to the nearest whole number, would be required for a given amount of this isotope to decay to 75% of that amount?

$$0.75 = 1e^{-\frac{\ln 2}{230}t} \Rightarrow \ln 0.75 = -\frac{\ln 2}{230}t \Rightarrow t = \frac{-230 \ln 0.75}{\ln 2} \approx 95 \text{ years}$$

There is 1 problem like #6 on the test.

6) $10000 = 1000\left(1 + \dfrac{0.0725}{4}\right)^{4t} \Rightarrow \ln 10 = \ln\left(1.018125^{4t}\right) \Rightarrow t = \dfrac{\ln 10}{4 \ln 1.018125} \approx 32.05$ years

There is 1 problem like #7 on the test.

7) The logistic differential equation $\dfrac{dP}{dt} = 0.034P\left(200 - P\right)$ describes the growth of a population P, where t is measured in years.

A) What is the carrying capacity of the population? 200

B) What is the size of the population when it is growing the fastest? 100

C) What is the rate at which the population is growing when it is growing the fastest? 340