

$$1) \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$2) \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$$

$$3) \text{ a) } \frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$$

$$\text{b) } \frac{dV}{dt} = 2\pi r \frac{dr}{dt} h$$

$$\text{c) We must apply the Product Rule: } \frac{dV}{dt} = \pi r^2 \frac{dh}{dt} + 2\pi r \frac{dr}{dt} h$$

$$4) \text{ a) } \frac{dP}{dt} = R \left(2I \frac{dI}{dt} \right) + \frac{dR}{dt} I^2 = 2RI \frac{dI}{dt} + I^2 \frac{dR}{dt}$$

$$\text{b) } 0 = R \left(2I \frac{dI}{dt} \right) + \frac{dR}{dt} I^2 \Rightarrow \frac{dR}{dt} = \frac{-2RI \frac{dI}{dt}}{I^2} = -\frac{2R}{I} \frac{dI}{dt}$$

$$5) \frac{ds}{dt} = \frac{1}{2} \left(x^2 + y^2 + z^2 \right)^{-1/2} \left(2x \frac{dx}{dt} + 2y \frac{dy}{dt} + 2z \frac{dz}{dt} \right) = \frac{x \frac{dx}{dt} + y \frac{dy}{dt} + z \frac{dz}{dt}}{\sqrt{x^2 + y^2 + z^2}}$$

$$6) A = \left(\frac{1}{2} ab \right) \sin \theta$$

$$\frac{dA}{dt} = \left(\frac{1}{2} ab \right) \cos \theta \frac{d\theta}{dt} + \left(\frac{1}{2} a \frac{db}{dt} + \frac{1}{2} b \frac{da}{dt} \right) \sin \theta = \frac{1}{2} \left(ab \cos \theta \frac{d\theta}{dt} + a \sin \theta \frac{db}{dt} + b \sin \theta \frac{da}{dt} \right)$$

$$7) \text{ a) } 1 \text{ volt/sec}$$

$$\text{b) } -1/3 \text{ amp/sec}$$

$$\text{c) } \frac{dV}{dt} = I \frac{dR}{dt} + R \frac{dI}{dt}$$

$$\text{d) } 12 = 1R \Rightarrow R = 12 \quad 1 = 2 \left(\frac{dR}{dt} \right) + 12 \left(-\frac{1}{3} \right) \Rightarrow \frac{dR}{dt} = \frac{5}{2} \text{ ohms/sec. Increasing, as } dR/dt \text{ is pos.}$$

$$8) \frac{dA}{dt} = 2\pi r \frac{dr}{dt} \quad \frac{dA}{dt} = 2\pi(50)(0.01) = \pi \text{ cm}^2/\text{sec}$$

$$9) \text{ a) } \frac{dA}{dt} = l \frac{dw}{dt} + w \frac{dl}{dt} \Rightarrow \frac{dA}{dt} = 12(2) + 5(-2) = 14 \text{ cm}^2/\text{sec}$$

$$\text{b) } \frac{dP}{dt} = 2 \frac{dl}{dt} + 2 \frac{dw}{dt} \Rightarrow \frac{dP}{dt} = 2(-2) + 2(2) = 0 \text{ cm/sec}$$

$$\text{c) } d = \sqrt{l^2 + w^2} \Rightarrow \frac{dd}{dt} = \frac{l \frac{dl}{dt} + w \frac{dw}{dt}}{\sqrt{l^2 + w^2}} = \frac{12(-2) + 5(2)}{\sqrt{144 + 25}} = \frac{-14}{13} \text{ cm/sec}$$

10) a) $V = x(yz) \Rightarrow \frac{dV}{dt} = x\left(y\frac{dz}{dt} + z\frac{dy}{dt}\right) + \frac{dx}{dt}(yz) = xy\frac{dz}{dt} + xz\frac{dy}{dt} + yz\frac{dx}{dt}$

$$\frac{dV}{dt} = 4(3)(1) + 4(2)(-2) + 3(2)(1) = 2 \text{ m}^3/\text{sec}$$

b) $S = 2xy + 2yz + 2xz \Rightarrow \frac{dS}{dt} = 2x\frac{dy}{dt} + 2y\frac{dx}{dt} + 2y\frac{dz}{dt} + 2z\frac{dy}{dt} + 2x\frac{dz}{dt} + 2z\frac{dx}{dt}$
 $\frac{dS}{dt} = 2(4)(-2) + 2(3)(1) + 2(3)(1) + 2(2)(-2) + 2(4)(1) + 2(2)(1) = 0 \text{ m}^2/\text{sec}$

c) $\frac{ds}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt} + z\frac{dz}{dt}}{\sqrt{x^2 + y^2 + z^2}}$ from problem #5

$$\frac{ds}{dt} = \frac{4(1) + 3(-2) + 2(1)}{\sqrt{16 + 9 + 4}} = \frac{0}{\sqrt{29}} = 0 \text{ m/sec}$$

11) $V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

a) $100\pi = 4\pi(5^2)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = 1 \text{ ft/min}$

b) $S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt}$

$$\frac{dS}{dt} = 8\pi(5)(1) = 40\pi \text{ ft}^2/\text{min}$$

12) $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} \quad S = 4\pi r^2$

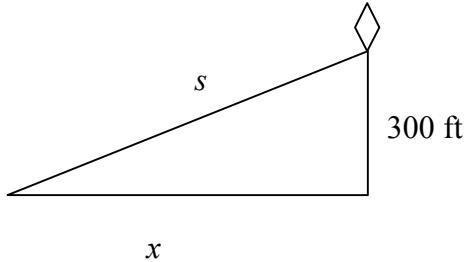
"picks up moisture at a rate proportional to its surface area" $\Rightarrow \frac{dV}{dt} = kS$

$$4\pi r^2 \frac{dr}{dt} = kS \Rightarrow 4\pi r^2 \frac{dr}{dt} = k4\pi r^2 \Rightarrow \frac{dr}{dt} = k$$

13) $s^2 = x^2 + 49 \Rightarrow 2s\frac{ds}{dt} = 2x\frac{dx}{dt} + 0 \Rightarrow \frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$

$$\frac{dx}{dt} = \frac{10}{\sqrt{51}}(300) \approx 420.084 \text{ mph}$$

14)



$$s^2 = x^2 + 90,000 \Rightarrow 2s \frac{ds}{dt} = 2x \frac{dx}{dt} + 0 \Rightarrow \frac{ds}{dt} = \frac{x}{s} \frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{400}{500}(25) = 20 \text{ ft/sec}$$

15)

$$V = \pi r^2 h \Rightarrow \frac{dV}{dt} = (\pi r^2) \frac{dh}{dt} + 2\pi r \frac{dr}{dt} (h)$$

$$\frac{dV}{dt} = \pi(1.9^2)(0) + 2\pi(1.9)\left(\frac{0.001}{3}\right)(6) \approx 0.0239 \text{ in}^3/\text{min.}$$

16a)

$$V = \frac{1}{3}\pi r^2 h \quad h = \frac{3}{4}r \Rightarrow r = \frac{4}{3}h$$

$$\text{To find } \frac{dh}{dt}: \quad V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h = \frac{16}{27}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{16}{9}\pi h^2 \frac{dh}{dt}$$

$$10 = \frac{16}{9}\pi(4)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{90}{256\pi} \approx 0.1119 \text{ m/min} = 11.19 \text{ cm/min}$$

$$\text{To find } \frac{dr}{dt}: \quad V = \frac{1}{3}\pi r^2 \left(\frac{3}{4}r\right) = \frac{1}{4}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{3}{4}\pi r^2 \frac{dr}{dt}$$

$$10 = \frac{3}{4}\pi\left(\frac{16}{3}\right)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{30}{64\pi} \approx 0.1492 \text{ m/min} = 14.92 \text{ cm/min}$$

16b)

$$V = \frac{1}{3}\pi r^2 h \quad h = \frac{3}{4}r \Rightarrow r = \frac{4}{3}h$$

$$\text{To find } \frac{dh}{dt}: \quad V = \frac{1}{3}\pi\left(\frac{4}{3}h\right)^2 h = \frac{16}{27}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{16}{9}\pi h^2 \frac{dh}{dt}$$

$$10 = \frac{16}{9}\pi(4)^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{90}{256\pi} \approx 0.1119 \text{ m/min} = 11.19 \text{ cm/min}$$

$$\text{To find } \frac{dr}{dt}: \quad V = \frac{1}{3}\pi r^2 \left(\frac{3}{4}r\right) = \frac{1}{4}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{3}{4}\pi r^2 \frac{dr}{dt}$$

$$10 = \frac{3}{4}\pi\left(\frac{16}{3}\right)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{30}{64\pi} \approx 0.1492 \text{ m/min} = 14.92 \text{ cm/min}$$

17) $V = \frac{1}{3}\pi r^2 h$ $\frac{DV}{dt} = -50$ $\frac{r}{h} = \frac{45}{6} \Rightarrow r = \frac{15}{2}h \Rightarrow h = \frac{2}{15}r$

To find $\frac{dh}{dt}$: $V = \frac{1}{3}\pi\left(\frac{15}{2}h\right)^2 h = \frac{75}{4}\pi h^3 \Rightarrow \frac{dV}{dt} = \frac{225}{4}\pi h^2 \frac{dh}{dt}$

$$-50 = \frac{225}{4}\pi(5)^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{-8}{225\pi} \approx -0.0113 \text{ m/min} = -1.13 \text{ cm/min}$$

To find $\frac{dr}{dt}$: $V = \frac{1}{3}\pi r^2 \left(\frac{2}{15}r\right) = \frac{2}{45}\pi r^3 \Rightarrow \frac{dV}{dt} = \frac{2}{15}\pi r^2 \frac{dr}{dt}$

$$-50 = \frac{2}{15}\pi\left(\frac{75}{2}\right)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{-4}{15\pi} \approx -0.0849 \text{ m/min} = -8.49 \text{ cm/min}$$

18) $V = \frac{\pi}{3}y^2(39-y) = 13\pi y^2 - \frac{\pi}{3}y^3$ $\frac{DV}{dt} = -6$

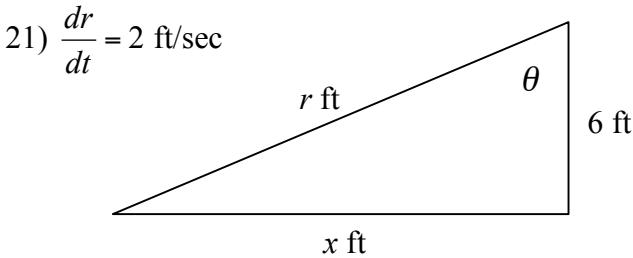
a) To find $\frac{dy}{dt}$: $\frac{dV}{dt} = 26\pi y \frac{dy}{dt} - \pi y^2 \frac{dy}{dt}$

$$-6 = 26\pi(8) \frac{dy}{dt} - \pi(64) \frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{-6}{208\pi - 64\pi} \approx -0.0132 \text{ m/min} = -1.32 \text{ cm/min}$$

b) To find r : $r^2 + (13-y)^2 = 13^2 \Rightarrow r = \sqrt{169 - (13-y)^2} = \sqrt{26y - y^2}$

c) $\frac{dr}{dt} = \frac{26-2y}{2\sqrt{26y-y^2}} \frac{dy}{dt} = \frac{13-y}{\sqrt{26y-y^2}} \frac{dy}{dt}$

when $y = 8$: $\frac{dr}{dt} = \frac{13-8}{\sqrt{26(8)-8^2}}(-0.0132) \approx -0.0055 \text{ m/min} = -0.55 \text{ cm/min}$



a) Find $\frac{dx}{dt}$ when $r = 10$

b) Find $\frac{d\theta}{dt}$ when $r = 10$
 $\cos\theta = 0.6 \Rightarrow \theta \approx 0.927$

$x = 8 \quad \frac{dy}{dt} = 0$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = r \frac{dr}{dt}$$

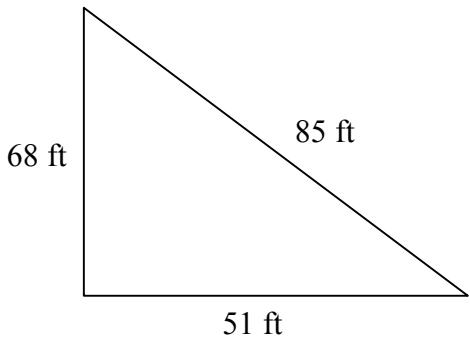
$$8 \frac{dx}{dt} + 6(0) = 10(2) \Rightarrow \frac{dx}{dt} = \frac{5}{2} \text{ ft/sec}$$

$\cos\theta = \frac{y}{r} \Rightarrow -\sin\theta \frac{d\theta}{dt} = \frac{r \frac{dy}{dt} - y \frac{dr}{dt}}{r^2}$

$$-\sin(0.927) \frac{d\theta}{dt} = \frac{10(0) - 6(-2)}{10^2}$$

$$\frac{d\theta}{dt} = \frac{0.12}{-\sin(0.927)} \approx -0.150 \text{ radians/sec}$$

22)



$$\frac{dx}{dt} = 17 \quad \frac{dy}{dt} = 1 \quad \text{Find } \frac{dc}{dt}$$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = c \frac{dc}{dt}$$

$$51(17) + 68(1) = 85 \frac{dc}{dt}$$

$$\frac{dc}{dt} = \frac{935}{85} = 11 \text{ ft/sec}$$

$$23) \quad \frac{dy}{dt} = \frac{-20x}{(1+x^2)^2} \frac{dx}{dt}$$

$$\text{a) } \frac{dy}{dt} = \frac{-20(-2)}{\left(1+(-2)^2\right)^2}(3) = \frac{120}{25} = \frac{24}{5} \text{ cm/sec}$$

$$\text{b) } \frac{dy}{dt} = \frac{-20(0)}{\left(1+(0)^2\right)^2}(3) = 0 \text{ cm/sec}$$

$$\text{c) } \frac{dy}{dt} = \frac{-20(20)}{\left(1+(20)^2\right)^2}(3) \approx -0.00746 \text{ cm/sec}$$

$$24) \quad \frac{dy}{dt} = 3x^2 \frac{dx}{dt} - 4 \frac{dx}{dt}$$

$$\text{a) } \frac{dy}{dt} = 3(-3)^2(-2) - 4(-2) = -54 + 8 = -46 \text{ cm/sec}$$

$$\text{b) } \frac{dy}{dt} = 3(1)^2(-2) - 4(-2) = -6 + 8 = 2 \text{ cm/sec}$$

$$\text{c) } \frac{dy}{dt} = 3(4)^2(-2) - 4(-2) = -96 + 8 = -88 \text{ cm/sec}$$

$$25) \quad \frac{dy}{dt} = 2x \frac{dx}{dt} \quad \frac{dx}{dt} = 10 \quad x = 3 \quad \frac{dy}{dt} = 60 \text{ m/sec}$$

$$\tan \theta = \frac{y}{x}$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2}$$

$$10 \frac{d\theta}{dt} = \frac{3(60) - 9(10)}{9} \Rightarrow \frac{d\theta}{dt} = 1 \text{ radian/sec}$$

$$26) \quad \frac{dy}{dt} = \frac{-1}{2\sqrt{-x}} \frac{dx}{dt} \quad \frac{dx}{dt} = -8 \quad x = -4 \quad \frac{dy}{dt} = 2 \text{ m/sec}$$

$$\tan \theta = \frac{y}{x}$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2}$$

$$\frac{5}{4} \frac{d\theta}{dt} = \frac{-4(2) - 2(-8)}{16} \Rightarrow \frac{d\theta}{dt} = \frac{2}{5} \text{ radian/sec}$$

$$27) \quad \frac{dV}{dt} = -8 \quad r = 10 \quad \text{Find } \frac{dS}{dt}$$

$$S = 4\pi r^2 \Rightarrow \frac{dS}{dt} = 8\pi r \frac{dr}{dt} \quad \text{to find this, we need to find } \frac{dr}{dt}$$

$$V = \frac{4}{3}\pi r^3 \Rightarrow \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$-8 = 4\pi(10)^2 \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{-1}{50\pi}$$

$$\frac{dS}{dt} = 8\pi(10) \left(\frac{-1}{50\pi} \right) = -\frac{8}{5} \text{ cm}^2/\text{min}$$

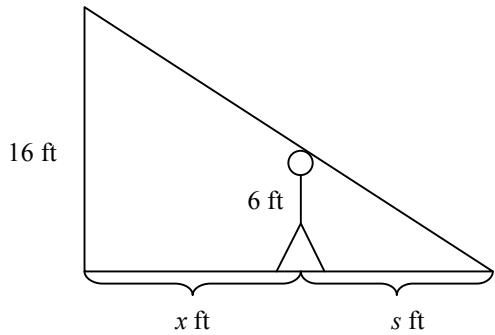
So the surface area of ice is decreasing at $\frac{8}{5} \text{ cm}^2/\text{min}$

$$28) \quad x \frac{dx}{dt} + y \frac{dy}{dt} = c \frac{dc}{dt} \quad \text{Find } \frac{dc}{dt}$$

$$5(-1) + 12(-5) = 13 \frac{dc}{dt}$$

$$\frac{dc}{dt} = \frac{-65}{13} = -5 \text{ m/sec}$$

29)



$$\text{Find } \frac{ds}{dt} \quad x = 10$$

$$\frac{s}{s+10} = \frac{6}{16} \Rightarrow 8s = 3x + 30 \Rightarrow s = 6$$

$$\frac{s}{x+s} = \frac{6}{16} \Rightarrow \frac{(x+s)\frac{ds}{dt} - s\left(\frac{dx}{dt} + \frac{ds}{dt}\right)}{(x+s)^2} = 0 \Rightarrow x\frac{ds}{dt} + s\frac{ds}{dt} - s\frac{dx}{dt} - s\frac{ds}{dt} = 0$$

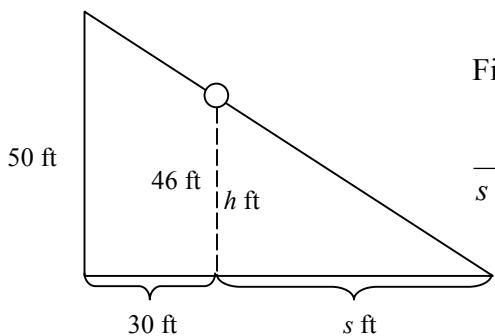
$$10\frac{ds}{dt} - 6(-5) = 0 \Rightarrow \frac{ds}{dt} = -3 \text{ ft/sec}$$

Having done it the hard way, let's see if we can do a little algebra to make it easier:

$$\frac{s}{x+s} = \frac{6}{16} \Rightarrow 16s = 6x + 6s \Rightarrow 10s = 6x \Rightarrow s = \frac{3}{5}x$$

$$\frac{ds}{dt} = \frac{3}{5} \frac{dx}{dt} = \frac{3}{5}(-5) = -3 \text{ ft/sec}$$

30)



$$\text{Find } \frac{ds}{dt} \quad h = 16t^2 \quad \frac{dh}{dt} = -32t$$

$$\frac{s}{s+30} = \frac{46}{50} \Rightarrow 25s = 23s + 690 \Rightarrow s = 345$$

$$\frac{s}{s+30} = \frac{h}{50} \Rightarrow 50s = sh + 30h$$

$$50\frac{ds}{dt} = s\frac{dh}{dt} + h\frac{ds}{dt} + 30\frac{dh}{dt}$$

$$50\frac{ds}{dt} = 345(-16) + 46\frac{ds}{dt} + 30(-16)$$

$$4\frac{ds}{dt} = -6000 \Rightarrow \frac{ds}{dt} = -1500 \text{ ft/sec}$$

31)

$$\tan \theta = \frac{y}{x} \quad \sec^2 \theta \frac{d\theta}{dt} = \frac{y \frac{dx}{dt} - x \frac{dy}{dt}}{y^2}$$

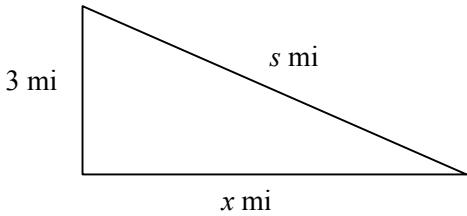
When the car is right in front of you, $\theta = 0$.

$$\sec^2(0) \frac{d\theta}{dt} = \frac{132(264) - 0(0)}{132^2} \Rightarrow \frac{d\theta}{dt} = 2 \text{ radians/sec}$$

$$\frac{1}{2} \text{ second later : } x = 132 \text{ ft} \quad \tan \theta = \frac{132}{132} = 1 \Rightarrow \theta = \frac{\pi}{4}$$

$$\sec^2\left(\frac{\pi}{4}\right) \frac{d\theta}{dt} = \frac{132(264) - 132(0)}{132^2} \Rightarrow 2 \frac{d\theta}{dt} = 2 \Rightarrow \frac{d\theta}{dt} = 1 \text{ radian/sec}$$

32)



$$s = 5 \quad x = 4 \quad \frac{ds}{dt} = -160 \quad \frac{dy}{dt} = 0 \quad \text{Find } \frac{dx}{dt}$$

$$x \frac{dx}{dt} + y \frac{dy}{dt} = s \frac{ds}{dt}$$

$$4 \frac{dx}{dt} + 3(0) = 5(-160) \Rightarrow \frac{dx}{dt} = -200 \text{ mph}$$

Since the plane is traveling in the same direction of the car, the car's speed = 200 - 120 = 80 mph.

33)

$$\tan \theta = \frac{y}{x} \quad c = 100 \text{ ft} \quad \frac{dy}{dt} = 0 \quad \tan \theta = \frac{4}{3} \Rightarrow \theta \approx 0.927 \quad 0.27^\circ/\text{min} = 0.00471 \text{ radians/min}$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2} \Rightarrow \sec^2(0.927)(0.00471) \approx \frac{60(0) - 80\left(\frac{dx}{dt}\right)}{3600}$$

$$47.0629 \approx -80 \frac{dx}{dt} \Rightarrow \frac{dx}{dt} \approx -0.588 \text{ ft/min} \approx -7.06 \text{ in/min}$$

The shadow's length is decreasing at approx. 7.1 in/min

$$34) \quad \tan \theta = \frac{y}{x} \quad x = 20 \text{ m} \quad \frac{dx}{dt} = 1 \text{ m/sec} \quad y = 10 \quad \frac{dy}{dt} = -2 \text{ m/sec} \quad \tan \theta = \frac{10}{20} \Rightarrow \theta = 0.463 \approx 26.565^\circ$$

$$\sec^2 \theta \frac{d\theta}{dt} = \frac{x \frac{dy}{dt} - y \frac{dx}{dt}}{x^2} \Rightarrow \sec^2(0.463) \frac{d\theta}{dt} \approx \frac{20(-2) - 10(1)}{400}$$

$$1.249 \frac{d\theta}{dt} \approx \frac{-1}{8} \Rightarrow \frac{d\theta}{dt} \approx -0.100 \text{ radians/sec} \approx -5.734^\circ/\text{min}$$

The angle is decreasing at approx. 6° per min

$$35) \quad x^2 = a^2 + b^2 - 2ab\cos\theta \quad \cos\theta \text{ is constant: } -0.5, \text{ so } x^2 = a^2 + b^2 + ab$$

$$2x \frac{dx}{dt} = 2a \frac{da}{dt} + 2b \frac{db}{dt} + a \frac{db}{dt} + b \frac{da}{dt} \quad x = 7$$

$$2(7) \frac{dx}{dt} = 2(5)(14) + 2(3)(21) + 5(21) + 3(14) \Rightarrow \frac{dx}{dt} = 29.5 \text{ knots}$$