

Calculus Section 4.3 Connecting f' and f'' with the Graph of f

$$1) y' = 2x - 1 \quad 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$$

x	0	$\frac{1}{2}$	1
y'	-1	0	1

Local (also absolute) maximum of $-\frac{5}{4}$ at $x = \frac{1}{2}$

$$2) y' = -6x^2 + 12x \quad -6x^2 + 12x = 0 \Rightarrow -6x(x - 2) = 0 \Rightarrow x = 0, 2$$

x	-1	0	1	2	3
y'	-18	0	6	0	-18

Local minimum of -3 at $x = 0$, local maximum of 5 at $x = 2$. No absolute extrema.

$$3) y' = 8x^3 - 8x \quad 8x^3 - 8x = 0 \Rightarrow 8x(x^2 - 1) = 0 \Rightarrow x = 0, \pm 1$$

x	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2
y'	-48	0	3	0	-3	0	48

Local (and absolute) minimum of -1 at $x = -1$ and at $x = 1$, local maximum of 1 at $x = 0$.

$$4) y' = x\left(\frac{-1}{x^2}e^{1/x}\right) + e^{1/x} = \frac{-e^{1/x} + xe^{1/x}}{x} \quad \frac{-e^{1/x} + xe^{1/x}}{x} = 0 \Rightarrow -e^{1/x} + xe^{1/x} = 0 \Rightarrow x = \frac{e^{1/x}}{e^{1/x}} = 1$$

x	0	1	2
y'	$\frac{-e^{1/x} + xe^{1/x}}{x}$	0	48

Local (and absolute) minimum of -1 at $x = -1$ and at $x = 1$, local maximum of 1 at $x = 0$.

$$5) \text{ Domain of } f: [-\sqrt{8}, \sqrt{8}]$$

$$y' = x\left(\frac{-x}{\sqrt{8-x^2}}\right) + 1\sqrt{8-x^2} = \frac{-x^2 + 8 - x^2}{\sqrt{8-x^2}} = \frac{8-2x^2}{\sqrt{8-x^2}} \quad \frac{8-2x^2}{\sqrt{8-x^2}} = 0 \Rightarrow x = \pm 2, \text{ and is only defined for } (-\sqrt{8}, \sqrt{8})$$

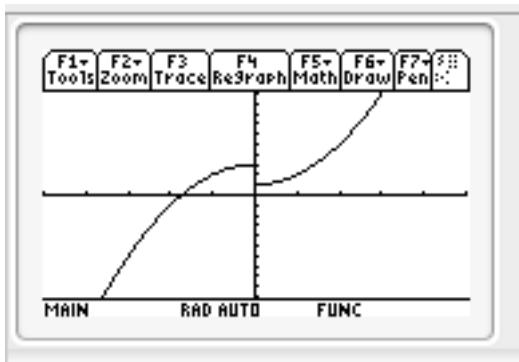
x	$-\sqrt{8}$	-2.1	-2	0	2	2.1	$\sqrt{8}$
y'	DNE	-3.979	0	$\frac{8}{\sqrt{8}}$	0	3.979	DNE

Local (and absolute) minimum of -4 at $x = -2$, local minimum of 0 at $x = \sqrt{8}$, local maximum of 0 at $x = -\sqrt{8}$, local (and absolute) maximum of 4 at $x = 2$.

6) $y' = \begin{cases} -2x & x < 0 \\ 2x & x > 0 \end{cases}$ y' is undefined for $x = 0$

x	-2	0	2
y'	4	DNE	4

Local minimum of 1 at $x = 0$.



7)

$$y' = 12x^2 + 42x + 36 \quad y'' = 24x + 42 \quad 24x + 42 = 0 \Rightarrow x = -\frac{7}{4}$$

$$y''(-2) = -6 \quad y''(-1) = 18$$

- a) Concave up on $\left(-\frac{7}{4}, \infty\right)$ b) concave down on $\left(-\infty, -\frac{7}{4}\right)$

8)

$$y' = -4x^3 + 12x^2 - 4 \quad y'' = -12x^2 + 24x \quad -12x^2 + 24x = 0 \Rightarrow -12x(x - 2) = 0 \Rightarrow x = 0, 2$$

$$y''(-1) = -36 \quad y''(1) = 12 \quad y''(3) = -36$$

- a) Concave up on $(0, 2)$ b) concave down on $(-\infty, 0), (2, \infty)$

9)

$$y' = \frac{2}{5}x^{-4/5} \quad y'' = -\frac{8}{25}x^{-9/5} = \frac{-8}{25x^{9/5}} \quad y'' \text{ never equals 0, but is undefined at } x = 0$$

$$y''(-1) = \frac{8}{25} \quad y''(1) = \frac{-8}{25}$$

- a) Concave up on $(-\infty, 0)$ b) concave down on $(0, \infty)$

10)

$$y' = -\frac{1}{3}x^{-2/3} \quad y'' = \frac{2}{9}x^{-5/3} = \frac{2}{9x^{5/3}} \quad y'' \text{ never equals 0, but is undefined at } x = 0$$

$$y''(-1) = -\frac{2}{9} \quad y''(1) = \frac{2}{9}$$

- a) Concave up on $(0, \infty)$ b) concave down on $(-\infty, 0)$

11)

$$y' = \begin{cases} 2 & x < 1 \\ -2x & x > 1 \end{cases} \quad y'' = \begin{cases} 0 & x < 1 \\ -2 & x > 1 \end{cases}$$

- a) Concave up nowhere b) concave down on $(1, \infty)$

12)

$$y' = e^x \quad y'' = e^x \quad y'' \text{ never equals } 0$$

$$y''(1) = e$$

- a) Concave up on $(0, 2\pi)$ b) concave down nowhere

13)

$$y' = xe^x + e^x \quad y'' = xe^x + e^x + e^x = xe^x + 2e^x = e^x(x + 2)$$

$$e^x(x + 2) = 0 \Rightarrow x = -2 \quad \text{Pt. of inflection at } \left(-2, \frac{-2}{e^2}\right)$$

14)

$$y' = x \frac{-x}{\sqrt{9-x^2}} + \sqrt{9-x^2} = \frac{9-2x^2}{\sqrt{9-x^2}}$$

$$y'' = \frac{-4x\sqrt{9-x^2} - (9-2x^2)\left(\frac{-x}{\sqrt{9-x^2}}\right)}{9-x^2} = \frac{-4x(9-x^2) + x(9-2x^2)}{(9-x^2)^{3/2}} = \frac{-27x+2x^3}{(9-x^2)^{3/2}}$$

$$\frac{-27x+2x^3}{(9-x^2)^{3/2}} = 0 \Rightarrow x = 0, \pm\sqrt{\frac{27}{2}} \quad \text{Pt. of inflection at } (0, 0) \quad \left[\pm\sqrt{\frac{27}{2}} \text{ is out of the domain of } y \right]$$

15)

$$y' = \frac{1}{1+x^2} \quad y'' = \frac{-2x}{(1+x^2)^2}$$

$$\frac{-2x}{(1+x^2)^2} = 0 \Rightarrow x = 0 \quad \text{Pt. of inflection at } (0, 0)$$

16)

$$y' = 12x^2 - 4x^3 \quad y'' = 24x - 12x^2$$

$$24x - 12x^2 = 0 \Rightarrow 12x(2-x) = 0 \Rightarrow x = 0, 2 \quad \text{Pts. of inflection at } (0, 0), (2, 16)$$

17)

$$y' = \frac{4}{3}x^{1/3} - \frac{4}{3}x^{-2/3} \quad y'' = \frac{4}{9}x^{-2/3} + \frac{8}{9}x^{-5/3} = \frac{4x+8}{9x^{5/3}}$$

$$\frac{4x+8}{9x^{5/3}} = 0 \Rightarrow x = -2, y'' \text{ is undefined at } x = 0 \quad \text{Pts. of inflection at } (-2, 6\sqrt[3]{2}), (0, 0)$$

18)

$$y' = \frac{3}{2}x^{1/2} + \frac{3}{2}x^{-1/2} \quad y'' = \frac{3}{4}x^{-1/2} - \frac{3}{4}x^{-3/2} = \frac{3x-3}{4x^{3/2}}$$

$$\frac{3x-3}{4x^{3/2}} = 0 \Rightarrow x = 1 \quad \text{Pt. of inflection at } (1,4)$$

19)

$$y' = \frac{(x-2)(3x^2-4x+1) - (x^3-2x^2+x-1)(1)}{(x-2)^2} = \frac{2x^3-8x^2+8x-1}{(x-2)^2}$$

$$y'' = \frac{(x-2)^2(6x^2-16x+8) - (2x^3-8x^2+8x-1)(2x-4)}{(x-2)^4} = \frac{2x^3-12x^2+24x-14}{(x-2)^3}$$

$$\frac{2x^3-12x^2+24x-14}{(x-2)^3} = 0 \Rightarrow 2x^3-12x^2+24x-14 = 0 \Rightarrow x = 1 \quad \text{Pt. of inflection at } (1,1)$$

20)

$$y' = \frac{(x^2+1)(1) - x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$$

$$y'' = \frac{(x^2+1)^2(-2x) - (1-x^2)(4x^3+4x)}{(x^2+1)^4} = \frac{-2x^3-2x-4x+4x^3}{(x^2+1)^3} = \frac{2x^3-6x}{(x^2+1)^3}$$

$$\frac{2x^3-6x}{(x^2+1)^3} = 0 \Rightarrow 2x^3-6x = 0 \Rightarrow x = 0, \pm\sqrt{3} \quad \text{Pts. of inflection at } (0,0), \left(-\sqrt{3}, \frac{-\sqrt{3}}{4}\right), \left(\sqrt{3}, \frac{\sqrt{3}}{4}\right)$$

21) a) f' is 0 : $x = \pm 1$ f' is positive : $(-\infty, -1), (1, \infty)$ f' is negative : $(-1, 1)$

b) f'' is 0 : $(0, 0)$ f'' is positive : $(0, \infty)$ f'' is negative : $(-\infty, 0)$

22) a) f' is 0 : $x = 0, x \approx \pm 1.25$ f' is positive : $(-1.25, 0), (1.25, \infty)$ f' is negative : $(-\infty, -1.25), (0, 1.25)$

b) f'' is 0 : $x \approx \pm 0.75$ f'' is positive : $(-\infty, -0.75), (0.75, \infty)$ f'' is negative : $(-0.75, 0.75)$

23) a) $(-\infty, -2], [0, 2]$

b) $[-2, 0], [2, \infty)$

c) Local max at $x = -2$ and $x = 2$; local min at $x = 0$

24) a) $[-2, 2]$

b) $[-\infty, -2], [2, \infty)$

c) Local max at $x = 2$; local min at $x = -2$

25) a) $v(t) = 2t - 4$

b) $a(t) = 2$

c) The particle starts at $x = 3$, moves to the left for $0 \leq t \leq 2$ to $x = -1$, then moves to the right for $t \geq 2$

26) a) $v(t) = -2 - 2t$

b) $a(t) = -2$

c) The particle starts at $x = 6$ and moves to the left for $t \geq 0$

27) a) $v(t) = 3t^2 - 3$

b) $a(t) = 6t$

c) The particle starts at $x = 3$ and moves to the left for $0 \leq t \leq 1$ to $x = 1$, then moves to the right for $t \geq 1$

28) a) $v(t) = 6t - 6t^2$

b) $a(t) = 6 - 12t$

c) The particle starts at $x = 0$ and moves to the right for $0 \leq t \leq 1$ to $x = 1$, then moves to the left for $t \geq 1$

33)

$$y' = 3 - 3x^2 \quad y'' = -6x$$

$$3 - 3x^2 = 0 \Rightarrow x = \pm 1$$

$y''(-1) = 6$, so $(-1, 3)$ is a local min. $y''(1) = -6$, so $(1, 7)$ is a local max.

34)

$$y' = 5x^4 - 80 \quad y'' = 20x^3$$

$$5x^4 - 80 = 0 \Rightarrow x = \pm 2$$

$y''(-2) < 0$, so $(-2, 228)$ is a local max. $y''(2) > 0$, so $(2, -28)$ is a local min.

35)

$$y' = 3x^2 + 6x \quad y'' = 6x + 6$$

$$3x^2 + 6x = 0 \Rightarrow x = 0, -2$$

$y''(0) > 0$, so $(0, -2)$ is a local min. $y''(-2) < 0$, so $(-2, 2)$ is a local max.

36)

$$y' = 15x^4 - 75x^2 + 60 \quad y'' = 60x^3 - 150x$$

$$15x^4 - 75x^2 + 60 = 0 \Rightarrow 15(x^4 - 5x^2 + 4) = 0 \Rightarrow 15(x^2 - 4)(x^2 - 1) \Rightarrow x = \pm 2, \pm 1$$

$y''(-2) < 0$, so $(-2, 4)$ is a local max. $y''(-1) > 0$, so $(-1, -18)$ is a local min.

$y''(1) < 0$, so $(1, 58)$ is a local max. $y''(2) > 0$, so $(2, 36)$ is a local min.

37)

$$y' = xe^x + e^x \quad y'' = xe^x + 2e^x$$

$$xe^x + e^x = 0 \Rightarrow x = -1$$

$y''(-1) > 0$, so $(-1, -1/e)$ is a local min.

38)

$$y' = -xe^{-x} + e^{-x} \quad y'' = xe^{-x} - 2e^{-x}$$

$$-xe^{-x} + e^{-x} = 0 \Rightarrow x = 1$$

$y''(1) < 0$, so $(1, 1/e)$ is a local max.

39) $y' = 0$ when $x = 1$ or $x = 2$.

x	0	1	1.5	2	3
y'	-2	0	-0.125	0	4

a) no local max b) local min at $x = 2$

c)

$$y'' = (x-1)^2(1) + 2(x-1)(x-2) = x^2 - 2x + 1 + 2x^2 - 6x + 4 = 3x^2 - 8x + 5$$

$$3x^2 - 8x + 5 = 0 \Rightarrow (3x-5)(x-1) = 0 \Rightarrow x = 1, \frac{5}{3}$$

40) $y' = 0$ when $x = 1$ or $x = 2$ or $x = 4$.

x	0	1	1.5	2	3	4	5
y'	8	0	0.3125	0	-4	0	48

a) local max at $x = 2$ b) local min at $x = 4$

c)

$$y'' = (x-1)^2(2x-6) + 2(x-1)(x^2 - 6x + 8) = 4x^3 - 24x^2 + 42x - 22$$

$$4x^3 - 24x^2 + 42x - 22 = 0 \Rightarrow x = 0, \frac{5 \pm \sqrt{3}}{2}$$