Calculus Lesson 4.2 Mean Value Theorem

1) a) Yes, $f(x)$ is continuous on [0,1] and differentiable on (0,1)

b) $2c+2=\dfrac{2-(-1)}{1-0}\Rightarrow 2c+2=3\Rightarrow c=\dfrac{1}{2}$

2) a) Yes, $f(x)$ is continuous on [0,1] and differentiable on (0,1)

b) $\dfrac{2}{3c^{1/3}}=\dfrac{1-0}{1-0}\Rightarrow 3c^{1/3}=2\Rightarrow c^{1/3}=\dfrac{2}{3}\Rightarrow c=\dfrac{8}{27}$

3) a) No, there is a vertical asymptote at $x = 0$

4) a) No, there is a corner at $x = 1$

5) a) Yes, $f(x)$ is continuous on [-1,1] and differentiable on (-1,1)

b) $\dfrac{1}{\sqrt{1-c^2}}=\dfrac{\frac{\pi}{2}-\left(-\frac{\pi}{2}\right)}{1-(-1)}\Rightarrow\sqrt{1-c^2}=\dfrac{2}{\pi}\Rightarrow 1-c^2=\dfrac{4}{\pi^2}\Rightarrow c^2=1-\dfrac{4}{\pi^2}\Rightarrow c=\pm\sqrt{1-\dfrac{4}{\pi^2}}\approx\pm 0.771$

6) a) Yes, $f(x)$ is continuous on [2,4] and differentiable on (2,4)

b) $\dfrac{1}{c-1}=\dfrac{\ln 3-\ln 1}{3-1}\Rightarrow\dfrac{1}{c-1}=\dfrac{\ln 3}{2}\Rightarrow c-1=\dfrac{2}{\ln 3}\Rightarrow c=\dfrac{2+\ln 3}{\ln 3}\approx 2.820$

7) a) No, $f(x)$ has a discontinuity at $\dfrac{\pi}{2}$

8) a) No, $f(x)$ has a discontinuity at 1

9) a) Slope of line AB: $m=\dfrac{2.5-2.5}{2-0.5}=0$.

Equation: $y-2.5=0(x-2)$ or $y=2.5$

b) $1-\dfrac{1}{c^2}=0\Rightarrow c^2=1\Rightarrow c=1\quad f(1)=2\quad y-2=0(x-1)\Rightarrow y=2$

10) a) Slope of line AB: $m=\dfrac{\sqrt{2}-0}{3-1}=\dfrac{\sqrt{2}}{2}$.

Equation: $y-0=\dfrac{\sqrt{2}}{2}(x-1)$ or $y=\dfrac{\sqrt{2}}{2}x-\dfrac{\sqrt{2}}{2}$

b) $\dfrac{1}{2\sqrt{c-1}}=\dfrac{\sqrt{2}}{2}\Rightarrow\sqrt{2c-2}=1\Rightarrow 2c-2=1\Rightarrow c=\dfrac{3}{2}\quad f\left(\dfrac{3}{2}\right)=\dfrac{\sqrt{2}}{2}\quad y-\dfrac{\sqrt{2}}{2}=\dfrac{\sqrt{2}}{2}\left(x-\dfrac{3}{2}\right)\Rightarrow y=\dfrac{\sqrt{2}}{2}x-\dfrac{\sqrt{2}}{4}$

15) $f'(x) = 5 - 2x \quad 5 - 2x = 0 \Rightarrow x = \frac{5}{2}$

x	0	$\frac{5}{2}$	3
$f'(x)$	5	0	-1

The function increases on $\left(-\infty, \frac{5}{2}\right]$ and decreases on $\left[\frac{5}{2}, \infty\right)$

There is an absolute maximum value of $\frac{25}{4}$ at $x = \frac{5}{2}$

16) $f'(x) = 2x - 1 \quad 2x - 1 = 0 \Rightarrow x = \frac{1}{2}$

x	0	$\frac{1}{2}$	1
$f'(x)$	-1	0	1

The function decreases on $\left(-\infty, \frac{1}{2}\right]$ and increases on $\left[\frac{1}{2}, \infty\right)$

There is an absolute minimum value of $-\frac{49}{4}$ at $x = \frac{1}{2}$

17) $f'(x) = \frac{-2}{x^2} \quad \frac{-2}{x^2}$ never equals 0, but is undefined at $x = 0$

x	-1	0	1
$f'(x)$	-2	DNE	-2

The function decreases on $(-\infty, 0)$ and $(0, \infty)$

There are no extreme values

18) $f'(x) = \frac{-2}{x^3} \quad \frac{-2}{x^3}$ never equals 0, but is undefined at $x = 0$

x	-1	0	1
$f'(x)$	2	DNE	-2

The function increases on $(-\infty, 0)$ and decreases on $(0, \infty)$

There are no extreme values

19) $f'(x) = 2e^{2x} \quad 2e^{2x}$ never equals 0

The function increases on $(-\infty, \infty)$

There are no extreme values

20) $f'(x) = -0.5e^{-0.5x} \quad -0.5e^{-0.5x}$ never equals 0

The function decreases on $(-\infty, \infty)$

There are no extreme values

21) Domain of f: [-2,∞) $f'(x) = \frac{-1}{\sqrt{x+2}}$ $\frac{-1}{\sqrt{x+2}}$ never equals 0, but is undefined when $x \le -2$

x	-2	-1
$f'(x)$	DNE	-1

The function decreases on [-2,∞)
There is an absolute maximum value of 4 at x = -2

22) $f'(x) = 4x^3 - 20x$ $4x^3 - 20x = 0 \Rightarrow 4x\left(x^2 - 5\right) = 0 \Rightarrow x = 0, \pm\sqrt{5}$

x	-3	$-\sqrt{5}$	-1	0	1	$\sqrt{5}$	3
$f'(x)$	-48	0	16	0	-16	0	48

The function decreases on $\left(-\infty, -\sqrt{5}\right]$ and $\left[0, \sqrt{5}\right]$. The function increases on $\left[-\sqrt{5}, 0\right]$ and $\left[\sqrt{5}, \infty\right)$
There is an absolute minimum value of -16 at $x = -\sqrt{5}$ and at $x = \sqrt{5}$.
There is a local maximum of 9 when x = 0

23) Domain of f: (-∞,4]
$f'(x) = \frac{-x}{2\sqrt{4-x}} + \sqrt{4-x} = \frac{8-3x}{2\sqrt{4-x}}$ $\frac{8-3x}{2\sqrt{4-x}} = 0 \Rightarrow x = \frac{8}{3}$; $f'(x)$ is undefined when $x \ge 4$

x	-3	$-\sqrt{5}$	-1	0	1	$\sqrt{5}$	3
$f'(x)$	-48	0	16	0	-16	0	48

The function decreases on $\left(-\infty, -\sqrt{5}\right]$ and $\left[0, \sqrt{5}\right]$. The function increases on $\left[-\sqrt{5}, 0\right]$ and $\left[\sqrt{5}, \infty\right)$
There is an absolute minimum value of -16 at $x = -\sqrt{5}$ and at $x = \sqrt{5}$.
There is a local maximum of 9 when x = 0

24) $f'(x) = x^{1/3}(1) + \frac{x+8}{3x^{2/3}} = \frac{4x+8}{3x^{2/3}}$ $\frac{4x+8}{3x^{2/3}} = 0 \Rightarrow x = -2$; $f'(x)$ is undefined when $x = 0$

x	-3	- 2	-1	0	1
$f'(x)$	$\frac{-4\sqrt[3]{3}}{9}$	0	$\frac{4}{3}$	DNE	4

The function decreases on $\left(-\infty, -2\right]$. The function increases on $\left[-2, \infty\right)$
There is an absolute minimum value of $6\sqrt[3]{-2} \approx -7.560$ at x = -2.

25) $f'(x) = \dfrac{(x^2+4)(-1)-(-x)(2x)}{(x^2+4)^2} = \dfrac{3x^2-4}{(x^2+4)^2}$ $\quad \dfrac{3x^2-4}{(x^2+4)^2} = 0 \Rightarrow x = \pm\sqrt{\dfrac{4}{3}}$

x	-2	$-\sqrt{\frac{4}{3}}$	0	$\sqrt{\frac{4}{3}}$	2
$f'(x)$	$\frac{1}{8}$	0	$-\frac{1}{4}$	0	$\frac{1}{8}$

The function increases on $\left(-\infty,-\sqrt{\frac{4}{3}}\right]$ and $\left[\sqrt{\frac{4}{3}},\infty\right)$. The function decreases on $\left[-\sqrt{\frac{4}{3}},\sqrt{\frac{4}{3}}\right]$

There is a local minimum value of $-\frac{1}{4}$ at $x = 0$ and local maximum values of $\frac{1}{8}$ at $x = -2$ and $x = 2$.

26) $f'(x) = \dfrac{(x^2-4)(1)-(x)(2x)}{(x^2-4)^2} = \dfrac{-x^2-4}{(x^2-4)^2}$ $\quad \dfrac{-x^2-4}{(x^2-4)^2}$ never equals 0 and is undefined at $x = \pm 2$

x	-3	- 2	0	2	3
$f'(x)$	$-\frac{13}{25}$	DNE	$-\frac{1}{4}$	DNE	$-\frac{13}{25}$

The function decreases on $(-\infty,-2)$, $(-2,2)$, and $(2,\infty)$. The function never increases.
There are no extreme values.

27) $f'(x) = 3x^2 - 2 + 2\sin x$ $\quad 3x^2 - 2 + 2\sin x = 0 \Rightarrow x \approx -1.126, 0.559$

x	-2	- 1.126	0	0.559	1
$f'(x)$	8.181	0	-2	0	2.683

The function increases on $(-\infty,-1.126]$ and $[0.559,\infty)$ and decreases on $[-1.126,0.559]$.
There is a local maximum of approximately -0.036 at $x \approx -1.126$, and a local minimum of approximately -2.639 at $x \approx 0.559$.

28) $f'(x) = 2 - \sin x$ $\quad 2 - \sin x$ never equals 0

x	$-\pi/2$	0	$\pi/2$
$f'(x)$	3	2	1

The function increases on $(-\infty,\infty)$. There are no extreme values.