

Calculus Lesson 4.1 Extreme Values of Functions

11) $f'(x) = -\frac{1}{x^2} + \frac{1}{x} = \frac{x-1}{x^2}$. This equals 0 when $x = 1$

x	0.5	1	4
y	1.307	1	1.636

Local max at $(0.5, 1.307)$, local (and absolute) min at $(1, 1)$, local (and absolute) max at $(4, 1.636)$

12) $g'(x) = -e^{-x} = \frac{-1}{e^x}$. This never equals 0

x	-1	1
y	e	e^{-1}

Local (and absolute) max at $(-1, e)$, local (and absolute) min at $(1, e^{-1})$

13) $h'(x) = \frac{1}{x+1}$. This never equals 0

x	0	3
y	0	$\ln 4$

Local (and absolute) min at $(0, 0)$, local (and absolute) max at $(3, \ln 4)$

14) $k'(x) = -2xe^{-x^2} = \frac{-2x}{e^{x^2}}$. This equals 0 when $x = 0$. $k(0) = 1$

Local (and absolute) max at $(0, 1)$

15) $f'(x) = \cos\left(x + \frac{\pi}{4}\right)$. This equals 0 when $x + \frac{\pi}{4} = \frac{\pi}{2}$ or $\frac{3\pi}{2} \Rightarrow x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

x	0	$\frac{\pi}{4}$	$\frac{5\pi}{4}$	$\frac{7\pi}{4}$
y	$\frac{\sqrt{2}}{2}$	1	-1	0

Local min at $\left(0, \frac{\sqrt{2}}{2}\right)$, local (and absolute) max at $\left(\frac{\pi}{4}, 1\right)$, local (and absolute) min at $\left(\frac{5\pi}{4}, -1\right)$,

local max at $\left(\frac{7\pi}{4}, 0\right)$

16) $g'(x) = \sec x \tan x$. This equals 0 when $\sin x = 0 \Rightarrow x = 0, \pi$ on $\left[-\frac{\pi}{2}, \frac{3\pi}{2}\right]$ and is undefined at $x = \frac{\pi}{2}$

x	$-\frac{\pi}{2}$	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
y	DNE	1	DNE	-1	DNE

Local min at $(0, 1)$, local max at $(\pi, -1)$

17) $f'(x) = \frac{2}{5}x^{-3/5}$. This never equals 0, and is undefined when $x = 0$

x	-3	0
y	$(-3)^{2/5}$	0

Local (and absolute) min at $(0,0)$, local (and absolute) max at $(-3, (-3)^{2/5})$

18) $f'(x) = \frac{3}{5}x^{-2/5}$. This never equals 0, and is undefined when $x = 0$

x	0	3
y	0	$3^{3/5}$

Local (and absolute) max at $(3, 3^{3/5})$

$$19) y' = 4x - 8 \quad 4x - 8 = 0 \Rightarrow x = 2$$

x	1	2	3
y	3	1	3

Maximum value is 2 at $x = 1$

$$20) y' = 3x^2 - 2 \quad 3x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{\frac{2}{3}}$$

x	-1	$-\sqrt{\frac{2}{3}}$	0	$\sqrt{\frac{2}{3}}$	1
y	5	5.089	4	2.911	3

No absolute extremes. Local maximum value is approx. 5.089 at $x = -\sqrt{\frac{2}{3}}$, local minimum

value is approx. 2.911 at $x = \sqrt{\frac{2}{3}}$

$$21) y' = 3x^2 + 2x - 8 \quad 3x^2 + 2x - 8 = 0 \Rightarrow (3x - 4)(x + 2) = 0 \Rightarrow x = -2, \frac{4}{3}$$

x	-3	-2	0	$\frac{4}{3}$	2
y	11	17	5	$-\frac{41}{27}$	1

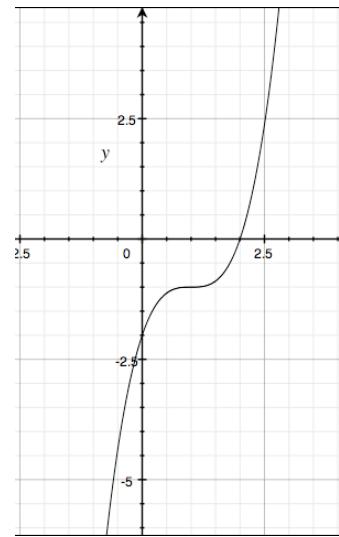
No absolute extremes. Local maximum value is 17 at $x = -2$, local minimum

value is $-\frac{41}{27}$ at $x = 2$

22) $y' = 3x^2 - 6x + 3 \quad 3x^2 - 6x + 3 = 0 \Rightarrow 3(x-1)(x-1) = 0 \Rightarrow x = 1$

x	0	1	2
y	-2	-1	0

No extremes, local or absolute.



23) $y' = \frac{x}{\sqrt{x^2 - 1}} \quad \frac{x}{\sqrt{x^2 - 1}} = 0 \Rightarrow x = 0$, and is undefined when $x = \pm 1$

The domain of y is $(-\infty, -1] \cup [1, \infty)$. Absolute minimum values of 0 when $x = \pm 1$

24) $y' = \frac{-2x}{(x^2 - 1)^2} \quad \frac{-2x}{(x^2 - 1)^2} = 0 \Rightarrow x = 0$, and is undefined when $x = \pm 1$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y	$-\frac{4}{3}$	-1	$-\frac{4}{3}$

Local maximum value of -1 when $x = 0$

25) Domain of y is $(-1, 1)$. $y' = \frac{x}{(1-x^2)^{3/2}} \quad \frac{x}{(1-x^2)^{3/2}} = 0 \Rightarrow x = 0$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y	$\frac{2}{\sqrt{3}}$	1	$\frac{2}{\sqrt{3}}$

Local minimum value of 1 when $x = 0$

26) Domain is all Reals except ± 1 . $y' = \frac{2x}{3(1-x^2)^{4/3}} \quad \frac{2x}{3(1-x^2)^{4/3}} = 0 \Rightarrow x = 0$

x	$-\frac{1}{2}$	0	$\frac{1}{2}$
y	$\sqrt[3]{\frac{4}{3}}$	1	$\sqrt[3]{\frac{4}{3}}$

Local minimum value of 1 when $x = 0$

27) Domain of the function is $[-1, 3]$. $y' = \frac{2-2x}{2\sqrt{3+2x-x^2}} = \frac{1-x}{\sqrt{3+2x-x^2}}$ $\frac{1-x}{\sqrt{3+2x-x^2}} = 0 \Rightarrow x = 1$

x	-1	0	1	2	3
y	0	$\sqrt{3}$	2	$\sqrt{3}$	0

Absolute maximum value of 2 at $x = 1$, absolute minimum value of 0 at $x = -1$ and $x = 3$.

28) $y' = 6x^3 + 12x^2 - 18x$ $6x^3 + 12x^2 - 18x = 0 \Rightarrow 6x(x+3)(x-1) = 0 \Rightarrow x = 0, 1, -3$

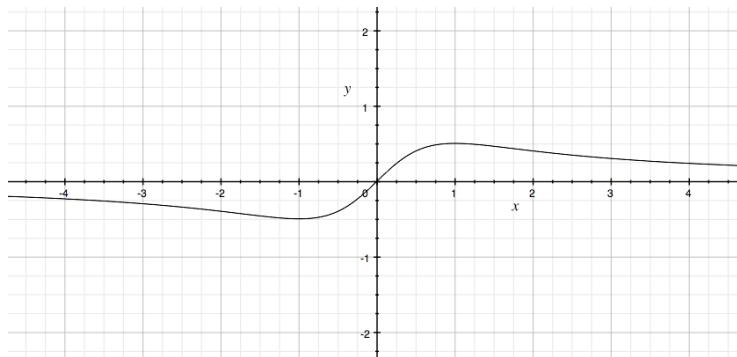
x	-4	-3	-1	0	$\frac{1}{2}$	1	2
y	-6	-57.5	-1.5	10	8.34375	6.5	30

Local (and absolute) min of -57.5 at $x = -3$, local max of 10 at $x = 0$, local min of 6.5 at $x = 1$

29) $y' = \frac{(x^2+1)(1)-x(2x)}{(x^2+1)^2} = \frac{1-x^2}{(x^2+1)^2}$ $\frac{1-x^2}{(x^2+1)^2} = 0 \Rightarrow 1-x^2 = 0 \Rightarrow x = \pm 1$

x	-2	-1	0	1	2
y	$-\frac{2}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{5}$

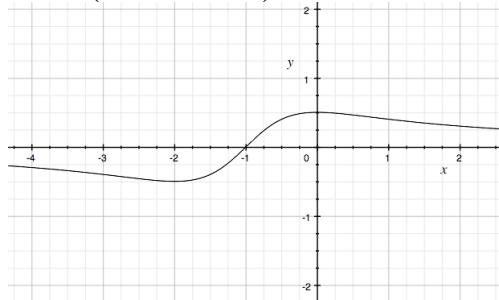
Local (and absolute) min of -0.5 at $x = -1$, local (and absolute) max of 0.5 at $x = 1$



30) $y' = \frac{(x^2+2x+2)(1)-(x+1)(2x+2)}{(x^2+2x+2)^2} = \frac{-x^2-2x}{(x^2+2x+2)^2}$ $\frac{-x^2-2x}{(x^2+2x+2)^2} = 0 \Rightarrow -x^2-2x=0 \Rightarrow x=0, -2$

x	-3	-2	-1	0	1
y	$-\frac{2}{5}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{2}{5}$

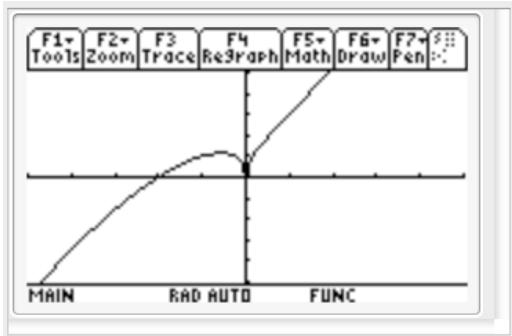
Local (and absolute) min of -0.5 at $x = -2$, local (and absolute) max of 0.5 at $x = 0$



$$35) y' = \frac{5}{3}x^{2/3} + \frac{4}{3}x^{-1/3} = \frac{5x+4}{3x^{1/3}} \quad \frac{5x+4}{3x^{1/3}} = 0 \Rightarrow x = -\frac{4}{5}, y' \text{ is undefined at } x = 0$$

x	-1	$-\frac{4}{5}$	$-\frac{2}{5}$	0	1
y	1	$\frac{12\sqrt[3]{10}}{25} \approx 1.034$	$\frac{8\sqrt[3]{20}}{25} \approx 0.869$	0	3

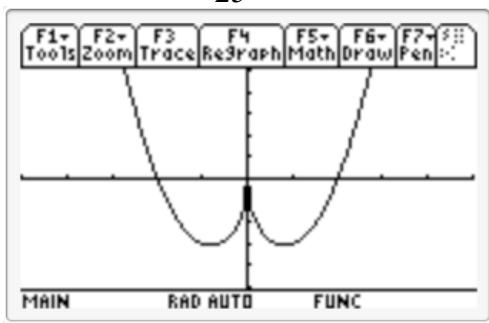
Local max of $\frac{12\sqrt[3]{10}}{25} \approx 1.034$ at $x = -0.8$, local min of 0 at $x = 0$



$$36) y' = \frac{8}{3}x^{5/3} - \frac{8}{3}x^{-1/3} = \frac{8x^2 - 8}{3x^{1/3}} \quad \frac{8x^2 - 8}{3x^{1/3}} = 0 \Rightarrow x = \pm 1, y' \text{ is undefined at } x = 0$$

x	-2	-1	$-\frac{2}{5}$	0	$\frac{2}{5}$	1	2
y	0	-3	$\frac{-96\sqrt[3]{20}}{125} \approx -2.085$	0	$\frac{-96\sqrt[3]{20}}{125} \approx -2.085$	-3	0

Local max of $\frac{12\sqrt[3]{10}}{25} \approx 1.034$ at $x = -0.8$, local min of 0 at $x = 0$

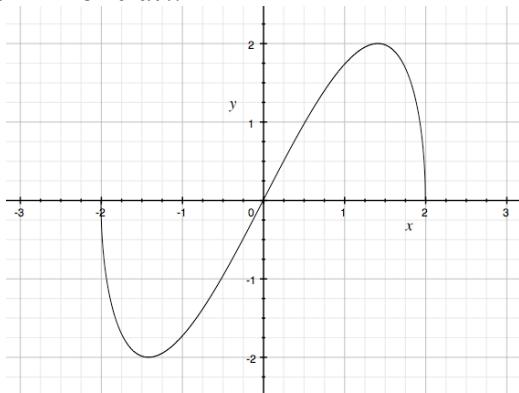


$$37) y' = x \left(\frac{-x}{\sqrt{4-x^2}} \right) + \sqrt{4-x^2} = \frac{4-2x^2}{\sqrt{4-x^2}} \quad \frac{4-2x^2}{\sqrt{4-x^2}} = 0 \Rightarrow x = \pm\sqrt{2}, y' \text{ is undefined at } x = \pm 2$$

The domain of y is $[-2, 2]$

x	-2	-1.5	$-\sqrt{2}$	0	$\sqrt{2}$	1.5	2
y	0	$\frac{-3\sqrt{7}}{4} \approx -1.984$	-2	0	2	$\frac{3\sqrt{7}}{4} \approx 1.984$	0

Local (and absolute) max of 2 at $x = \sqrt{2}$, local (and absolute) min of -2 at $x = -\sqrt{2}$, local max of 0 at $x = -2$, local min of 0 at $x = 2$

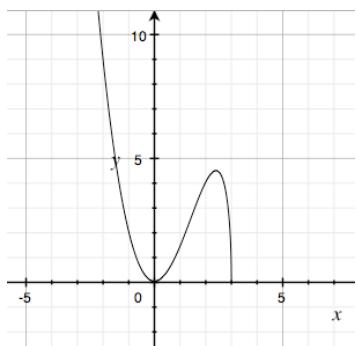


$$38) y' = x^2 \left(\frac{-1}{2\sqrt{3-x}} \right) + 2x\sqrt{3-x} = \frac{12x-5x^2}{\sqrt{3-x}} \quad \frac{12x-5x^2}{\sqrt{3-x}} = 0 \Rightarrow x = 0, \frac{12}{5}, y' \text{ is undefined at } x = 3$$

The domain of y is $(-\infty, 3]$

x	-1	0	1	$\frac{12}{5}$	$\frac{5}{2}$	3
y	2	0	$\sqrt{2}$	$\frac{144\sqrt{15}}{125} \approx 4.462$	$\frac{25\sqrt{2}}{8} \approx 4.419$	0

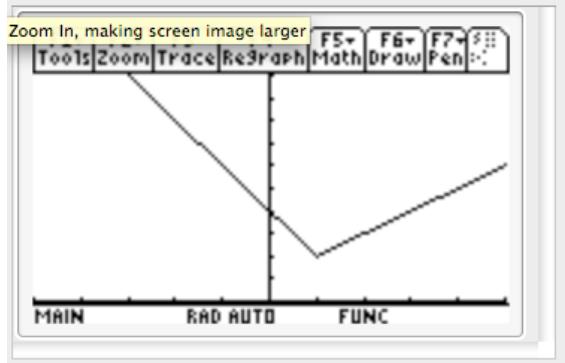
Local (and absolute) max of 2 at $x = \sqrt{2}$, local (and absolute) min of -2 at $x = -\sqrt{2}$, local max of 0 at $x = -2$, local min of 0 at $x = 2$



39) $y' = \begin{cases} -2 & x < 1 \\ 1 & x > 1 \end{cases}$ y' never equals 0, y' is undefined at $x = 1$

x	0	1	2
y	4	2	3

Local (and absolute) min of 2 at $x = 1$

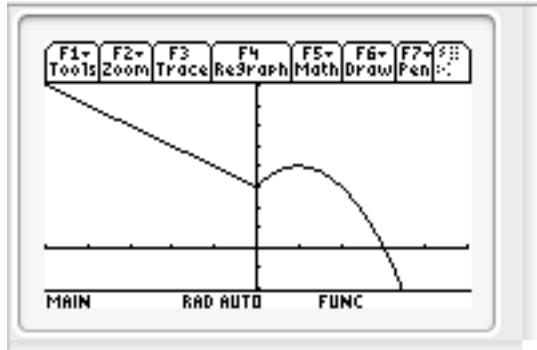


[-5,5] by [0,10]

40) $y' = \begin{cases} -1 & x < 0 \\ 2 - 2x & x > 0 \end{cases}$ $2 - 2x = 0 \Rightarrow y = 1$, y' is undefined at $x = 0$

x	-1	0	$\frac{1}{2}$	1	2
y	4	3	$\frac{15}{4}$	4	3

Local min of 3 at $x = 0$, local max of 4 at $x = 1$

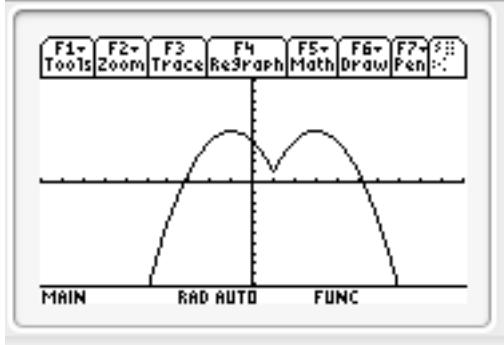


[-5,5] by [-2,8]

41) $y' = \begin{cases} -2x - 2 & x < 1 \\ -2x + 6 & x > 1 \end{cases}$ $-2x - 2 = 0 \Rightarrow x = -1$; $-2x + 6 = 0 \Rightarrow x = 3$; y' is undefined at $x = 1$

x	-2	-1	0	1	2	3	4
y	4	5	4	1	4	5	-4

Local min of 1 at $x = 1$, local (and absolute) max of 5 at $x = -1$ and $x = 3$



$[-10,10]$ by $[-10,10]$

42) $y' = \begin{cases} -\frac{1}{2}x - \frac{1}{2} & x < 1 \\ 3x^2 - 12x + 8 & x > 1 \end{cases}$ $-\frac{1}{2}x - \frac{1}{2} = 0 \Rightarrow x = -1$; $3x^2 - 12x + 8 = 0 \Rightarrow x = \frac{6+2\sqrt{3}}{3}$; y' is undefined at $x = 1$

x	-2	-1	0	1	$\frac{6+2\sqrt{3}}{3} \approx 3.155$	4
y	$\frac{15}{4}$	4	$\frac{15}{4}$	3	$-\frac{16\sqrt{3}}{9} \approx -3.079$	0

Local min of $-\frac{16\sqrt{3}}{9} \approx -3.079$ at $x = \frac{6+2\sqrt{3}}{3} \approx 3.155$, local max of 4 at $x = -1$

