What you'll learn about

- How *f'(a)* Might Fail to Exist
- Differentiability Implies Local Linearity
- Derivatives on a Calculator
- Differentiability Implies Continuity
- Intermediate Value Theorem for Derivatives

… and why

Graphs of differentiable functions can be approximated by their tangent lines at points where the derivative exists.

EQ: How does differentiability apply to the concepts of local linearity and continuity?

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How *f'(a)* Might Fail to Exist

A function will not have a derivative at a point $P(a, f(a))$ where the slopes of the secant lines, $\frac{f(x) - f(a)}{x - a}$ fail to approach a limit as x approaches a.

The next figures illustrate four different instances where this occurs.

For example, a function whose graph is otherwise smooth will fail to have a derivative at a point where the graph has:

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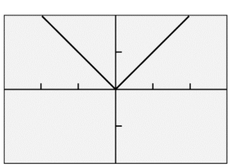
How *f'(a)* Might Fail to Exist

1. a corner, where the one-sided derivatives differ, as in $f(x) = |x|$



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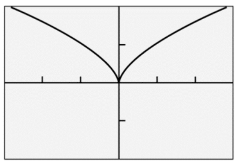
How *f'(a)* Might Fail to Exist

2. a cusp, where the slopes of the secant lines approach ∞ from one side and approach $-\infty$ from the other. This is an extreme case of a corner. $f(x) = x^{2/3}$ is an example of this.



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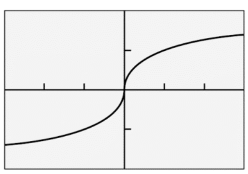
How *f'(a)* Might Fail to Exist

3. a vertical tangent, where the slopes of the secant lines approach either ∞ or $-\infty$ from both sides, as in $f(x) = x^{1/3}$.



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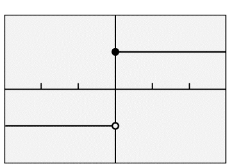
How *f'(a)* Might Fail to Exist

4. a discontinuity, which will cause one or both of the one-sided derivatives to be nonexistent.

$$f(x) = \begin{cases} -1, & x < 0 \\ 1, & x \ge 0 \end{cases}$$



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Example **How *f'(a)* Might Fail to Exist**

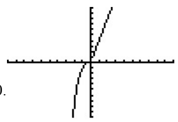
Show that the function is not differentiable at $x = 0$.

$$f(x) = \begin{cases} x^3, & x \le 0 \\ 4x, & x > 0 \end{cases}$$

The right-hand derivative is 4.

The left-hand derivative is 0.

The function is not differentiable at $x = 0$.



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How *f'(a)* Might Fail to Exist

Most of the functions we encounter in calculus are differentiable wherever they are defined, which means they will *not* have corners, cusps, vertical tangent lines or points of discontinuity within their domains. Their graphs will be unbroken and smooth, with a well-defined slope at each point.

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Differentiability Implies Local Linearity

A good way to think of differentiable functions is that they are **locally linear**; that is, a function that is differentiable at *a* closely resembles its own tangent line very close to *a*.

In the jargon of graphing calculators, differentiable curves will "straighten out" when we zoom in on them at a point of differentiability.

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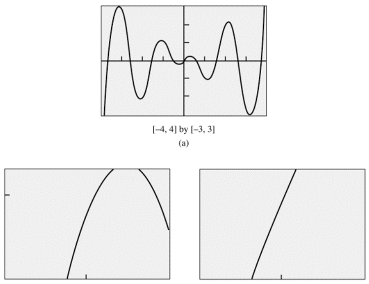
Differentiability Implies Local Linearity



[–4, 4] by [–3, 3]
(a)

[1.7, 2.3] by [1.7, 2.1]
(b)

[1.93, 2.07] by [1.85, 1.95]
(c)

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Derivatives on a Calculator

Many graphing utilities can approximate derivatives numerically with good accuracy at most points of their domains. For small values of h, the difference quotient $\frac{f(a+h) - f(a)}{h}$

is often a good numerical approximation of $f'(a)$.
However, the same value of h will usually yield a better approximation if we use the **symmetric difference quotient** $\frac{f(a+h) - f(a-h)}{2h}$

which is what our graphing calculator uses to calculate nDer $f(a)$, the **numerical derivative of f at a point a**.

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Example **Derivatives on a Calculator**

Find the numerical derivative of the function $f(x) = x^2 + 3$ at the point $x = 2$.

Using a TI-83 Plus we get

```
nDeriv(X^2+3,X,2
                4
█
```

Using a TI-89 we get

nDeriv(x^2+3,x)|x=2
4

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Derivatives on a Calculator

Because of the method used internally by the calculator, you will sometimes get a derivative value at a nondifferentiable point.

This is a case of where you must be "smarter" than the calculator.

Using a TI-89 , try

nDeriv(abs(x),x)|x=0

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Theorem 1: Differentiability Implies Continuity

If f has a derivative at $x = a$, then f is continuous at $x = a$.

The converse of Theorem 1 is false. A continuous function might have a corner, a cusp or a vertical tangent line, and hence not be differentiable at a given point.

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Theorem 2: Intermediate Value Theorem for Derivatives

Not every function can be a derivative.

If a and b are any two points in an interval on which f is differentiable, then f' takes on every value between $f'(a)$ and $f'(b)$.

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