What you'll learn about

- Definition of a Derivative
- Notation
- Relationship between the Graphs of f and f'
- Graphing the Derivative from Data
- One-sided Derivatives

… and why

The derivative gives the value of the slope of the tangent line to a curve at a point.

EQ: How do we find the derivative of a function?

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Definition of Derivative

The **derivative** of the function f with respect to the variable x is the function $f'(x)$, whose value at x is

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided the limit exists.

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Differentiable Function

The domain of f', the set of points in the domain of f for which the limit exists, may be smaller than the domain of f. If $f'(x)$ exists, we say that f **has a derivative (is differentiable)** at x. A function that is differentiable at every point in its domain is a **differentiable function**.

EQ: How do we find the derivative of a function?

Example **Definition of Derivative**

Differentiate $f(x) = x^2$

Applying the definition, we have

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{(x^2 + 2xh + h^2) - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} (2x + h)$$

$$= 2x$$

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EQ: How do we find the derivative of a function?

Derivative at a Point (alternate)

The derivative of the function f at the point where $x = a$ is the limit

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided the limit exists.

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EQ: How do we find the derivative of a function?

Notation

There are many ways to denote the derivative of a function $y = f(x)$. Besides $f'(x)$, the most common notations are:

y'	"y prime"	Nice and brief, but does not name the independent variable.
$\frac{dy}{dx}$	"dy dx" or "the derivative of y with respect to x"	Names both variables and uses d for derivative.
$\frac{df}{dx}$	"df dx" or "the derivative of f with respect to x"	Emphasizes the function's name.
$\frac{d}{dx} f(x)$	"d dx of f at x" or "the derivative of f at x"	Emphasis that differentiation is an operation performed on f.

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EQ: How do we find the derivative of a function?

Relationships between the Graphs of f and f'

Because we can think of the derivative at a point in graphical terms as slope, we can get a good idea of what the graph of the function f' looks like by *estimating the slopes* at various points along the graph of f.

We estimate the slope of the graph of f in y-units per x-unit at frequent intervals. We then plot the estimates in a coordinate plane with the horizontal axis in x-units and the vertical axis in slope units.

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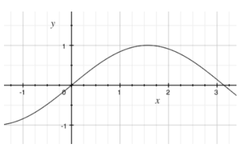
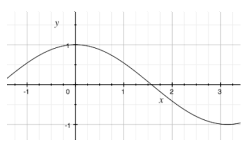
EQ: How do we find the derivative of a function?

Relationships between the Graphs of f and f'

The graph of a function f



The graph of the related function f'



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EQ: How do we find the derivative of a function?

Graphing the Derivative from Data

Discrete points plotted from sets of data do not yield a continuous curve, but we have seen that the shape and pattern of the graphed points (called a scatter plot) can be meaningful nonetheless. It is often possible to fit a curve to the points using regression techniques. If the fit is good, we could use the curve to get a graph of the derivative visually. However, it is also possible to get a scatter plot of the derivative numerically, directly from the data, by computing the slopes between successive points.

Now do an activity to illustrate this.

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One-sided Derivatives

A function $y = f(x)$ is **differentiable on a closed interval [*a*,*b*]** if it has a derivative at every interior point on the interval, and if the limits

$$\lim_{h \to 0^+} \frac{f(a+h) - f(a)}{h}$$ [the **right-hand derivative at *a***]

$$\lim_{h \to 0^-} \frac{f(a+h) - f(a)}{h}$$ [the **left-hand derivative at *a***]

exist at the endpoints. In the right-hand derivative, h is positive and $a + h$ approaches a from the right. In the left-hand derivative, h is negative and $b + h$ approaches b from the left.

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One-sided Derivatives

Right-hand and left-hand derivatives may be defined at any point of a function's domain.

The usual relationship between one-sided and two-sided limits holds for derivatives. Theorem 3, Section 2.1, allows us to conclude that a function has a (two-sided) derivative at a point if and only if the function's right-hand and left-hand derivatives are defined and equal at that point.

Example **One-sided Derivatives**

Show that the following function has left-hand and right-hand derivatives at $x = 0$, but no derivative there.

$$y = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

What is another way to express this function?

$y = |x|$!!!!!

Left-hand derivative:

$$\lim_{h \to 0^-} \frac{-(0+h) - 0}{h}$$

$$= \lim_{h \to 0^-} -\frac{h}{h} = -1$$

Right-hand derivative:

$$\lim_{h \to 0^+} \frac{(0+h) - 0}{h}$$

$$= \lim_{h \to 0^+} \frac{h}{h} = 1$$

The derivatives are not equal at $x = 0$. Therefore, the function does not have a derivative at 0.