1.4

Parametric Equations

What you'll learn about…

- Relations
- Circles
- Ellipses
- Lines and Other Curves

…and why

Parametric equations can be used to obtain graphs of relations and functions.

EQ:

What are parametric equations?

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Relations

- A **relation** is a set of ordered pairs (x, y) of real numbers.
- The **graph of a relation** is the set of points in a plane that correspond to the ordered pairs of the relation.
- If x and y are functions of a third variable t, called a *parameter*, then we can use the *parametric mode* of a grapher to obtain a graph of the relation.

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Parametric Curve, Parametric Equations

If x and y are given as functions
$x = f(t), y = g(t)$
over an interval of t-values, then the set of points
$(x, y) = (f(t), g(t))$
defined by these equations is a **parametric curve.**
The equations are **parametric equations** of the curve.

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Relations

The variable t is a parameter for the curve and its domain I is the parameter interval. If I is a closed interval, $a \le t \le b$, the point $(f(a), g(a))$ is the initial point of the curve and the point $(f(b), g(b))$ is the terminal point of the curve. When we give parametric equations and a parameter interval for a curve, we say that we have **parametrized** the curve. A grapher can draw a parametrized curve only over a closed interval, so the portion it draws has endpoints even when the curve being graphed does not.

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Example **Relations**

Describe the graph of the relation determined by $x = t, y = 1 - t^2$.

EXTENSION:

We can analyze this analytically by "removing the parameter". This process is where we solve for the parameter, then use substitution.

In the above problem, we would have $y = 1 - x^2$.

Note that this has the same graph as the parametric equation!

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Circles

- In applications, *t* often denotes time, an angle or the distance a particle has traveled along its path from a starting point.
- Parametric graphing can be used to simulate the motion of a particle.

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Example **Circles**

1. Describe the graph of the relation determined by $x = 3\cos t$, $y = 3\sin t$, $0 \le t \le 2\pi$
2. Find the initial points, if any, and indicate the direction in which the curve is traced.
3. Find a Cartesian equation for a curve that contains the parametrized curve.

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Ellipses

Parametrizations of ellipses are similar to parametrizations of circles. Recall that the standard form of an ellipse centered at (0,0) is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Suppose $x = a\cos t$ and $y = b\sin t$

For $x = a\cos t$ and $y = b\sin t$, we have

$\cos t = \frac{x}{a}$ and $\sin t = \frac{y}{b}$

From Trig: $\sin^2 x + \cos^2 x = 1$

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

which is the equation of an ellipse with center at the origin.

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Lines and Other Curves

- Lines, line segments and many other curves can be defined parametrically.

See Example 4, page 33.

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Example Lines and Other Curves

If the parametrization of a curve is $x = t$, $y = t + 2$, $0 \le t \le 2$, graph the curve.
2. Find the initial points, if any, and indicate the direction in which the curve is traced.
3. Find a Cartesian equation for a curve that contains the parametrized curve.
4. What portion of the Cartesion equation is traced by the parametrized curve?

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Lines and Other Curves

- Find the parametrization for the line segment with endpoints (3,-1) and (7,5).

- Initial point is (3,-1); terminal point is (7,5); assume $0 \le t \le 1$.
- $x = 3 + at$; $y = -1 + bt$
- $7 = 3 + a \Rightarrow a = 4$; $5 = -1 + b \Rightarrow b = 6$

- So, the parametrization is $x = 3 + 4t$, $y = -1 + 6t$, $0 \le t \le 1$.