What you'll learn about…

- Functions
- Domains and Ranges
- Viewing and Interpreting Graphs
- Even Functions and Odd functions - Symmetry
- Functions Defined in Pieces
- Absolute Value Function
- Composite Functions

…and why

Functions and graphs form the basis for understanding mathematics applications.

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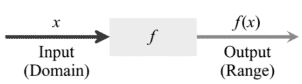
Functions

A rule that assigns to each element in one set a unique element in another set is called a *function.* A function is like a machine that assigns a unique output to every allowable input. The inputs make up the *domain* of the function; the outputs make up the *range*.



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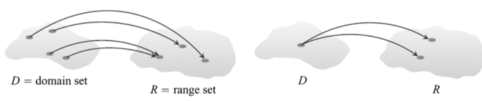
Function

A ***function*** from a set D to a set R is a rule that assigns a unique element in R to each element in D.

In this definition, D is the domain of the function and R is a set containing the range.



NOT A FUNCTION

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Function

The symbolic way to say "*y* is a function of *x*" is $y = f(x)$, which is read as "*y* equals *f* of *x*."

The notation $f(x)$ gives a way to denote specific values of a function. The value of *f* at *a* can be written as $f(a)$, read as "*f* of *a*."

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Example **Functions**

Evaluate the function $f(x) = 2x + 3$ when $x = 6$.

$f(6) = 2(6) + 3$

$f(6) = 12 + 3$

$f(6) = 15$

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Domains and Ranges

When we define a function $y = f(x)$ with a formula and the domain is not stated explicitly or restricted by context, the domain is assumed to be the largest set of *x*-values for which the formula gives real *y*-values -- the so-called natural domain. If we want to restrict the domain, we must say so.

The domain of $C(r) = 2\pi r$ is restricted by context, because the radius, *r*, must always be positive.

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Domains and Ranges

The domain of $y = 5x$ is assumed to be the entire set of real numbers.

If we want to restrict the domain of $y = 5x$ to be only positive values, we must write $y = 5x$, $x > 0$.

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Domains and Ranges

- The domains and ranges of many real-valued functions of a real variable are intervals or combinations of intervals. The intervals may be **open**, **closed** or **half-open, finite** or **infinite.**
- The endpoints of an interval make up the interval's **boundary** and are called **boundary points**.
- The remaining points make up the interval's **interior** and are called **interior points**.

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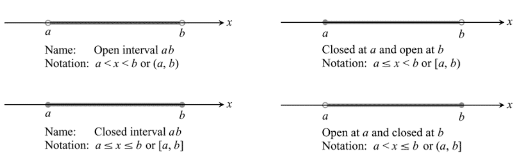
Domains and Ranges

- **Closed intervals** contain their boundary points.
- **Open intervals** contain no boundary points



Name: Open interval ab
Notation: $a < x < b$ or (a, b)

Closed at a and open at b
Notation: $a \le x < b$ or $[a, b)$

Name: Closed interval ab
Notation: $a \le x \le b$ or $[a, b]$

Open at a and closed at b
Notation: $a < x \le b$ or $(a, b]$

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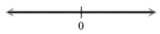
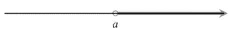
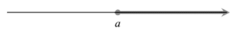
Domains and Ranges



Name: The set of all real numbers
Notation: $-\infty < x < \infty$ or $(-\infty, \infty)$



Name: The set of numbers greater than a
Notation: $a < x$ or (a, ∞)



Name: The set of numbers greater than or equal to a
Notation: $a \leq x$ or $[a, \infty)$

Name: The set of numbers less than b
Notation: $x < b$ or $(-\infty, b)$

Name: The set of numbers less than or equal to b
Notation: $x \leq b$ or $(-\infty, b]$

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Graph

The points (x, y) in the plane whose coordinates are the input-output pairs of a function $y = f(x)$ make up the function's **graph**.

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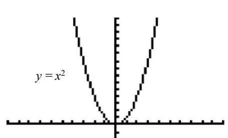
Example **Finding Domains and Ranges**

Identify the domain and range and use a grapher to graph the function $y = x^2$.

Domain: The function gives a real value of y for every value of x, so the domain is $(-\infty, \infty)$.
Range: Every value of the domain, x, gives a real, positive value of y, so the range is $[0, \infty)$.



[-10, 10] by [-5, 15]

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Viewing and Interpreting Graphs

Graphing with a graphing calculator requires that you develop graph *viewing* skills.

- Recognize that the graph is reasonable.
- See all the important characteristics of the graph.
- Interpret those characteristics.
- Recognize grapher failure.

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Viewing and Interpreting Graphs

Being able to recognize that a graph is reasonable comes with experience. You need to know the basic functions, their graphs, and how changes in their equations affect the graphs.

Grapher failure occurs when the graph produced by a grapher is less than precise – or even incorrect – usually due to the limitations of the screen resolution of the grapher.

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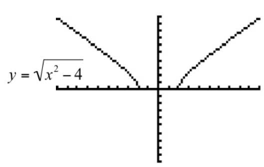
Example **Viewing and Interpreting Graphs**

Identify the domain and range and use a grapher to graph the function $y = \sqrt{x^2 - 4}$

Domain: The function gives a real value of y for each value of $|x| \geq 2$, so the domain is $(-\infty, -2]\cup[2,\infty)$.
Range: Every value of the domain, x, gives a real positive value of y, so the range is $[0,\infty)$.



[-10, 10] by [-10, 10]

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Even Functions and Odd Functions-Symmetry

- The graphs of *even* and *odd* functions have important symmetry properties.

A function $y = f(x)$ is a(n)
even function of *x* if $f(-x) = f(x)$
odd function of *x* if $f(-x) = -f(x)$
for every x in the function's domain.

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Even Functions and Odd Functions-Symmetry

- The graph of an **even** function is **symmetric about the *y*-axis**. A point (x,y) lies on the graph if and only if the point $(-x,y)$ lies on the graph.
- The graph of an **odd** function is **symmetric about the origin.** A point (x,y) lies on the graph if and only if the point $(-x,-y)$ lies on the graph.

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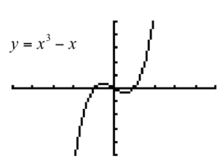
Example **Even Functions and Odd Functions-Symmetry**

Determine whether $y = x^3 - x$ is even, odd, or neither.

$y = x^3 - x$ is odd.
$f(-x) = (-x)^3 - (-x)$
$= -x^3 + x = -f(x)$



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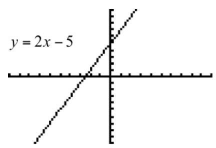
Example **Even Functions and Odd Functions-Symmetry**

Determine whether $y = 2x - 5$ is even, odd, or neither.

$y = 2x - 5$ is neither.
$f(-x) = 2(-x) - 5$
$= -2x - 5$



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Functions Defined in Pieces

- While some functions are defined by single formulas, others are defined by applying different formulas to different parts of their domain.
- These are called **piecewise functions**.

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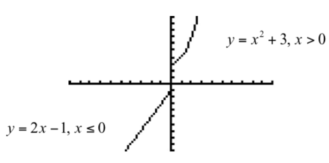
Example **Graphing a Piecewise Defined Function**

Graph the following piecewise function:

$$f(x) = \begin{cases} 2x - 1 & x \le 0 \\ x^2 + 3 & x > 0 \end{cases}$$



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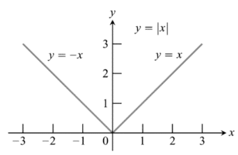
Absolute Value Functions

The absolute value function $y = |x|$ is defined piecewise by the formula

$$|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

The function is even, and its graph is symmetric about the y-axis



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Composite Functions

Suppose that some of the outputs of a function g can be used as inputs of a function f. We can then link g and f to form a new function whose inputs x are inputs of g and whose outputs are the numbers $f(g(x))$.

We say that the function $f(g(x))$, which is read "f of g of x", is the **composite of g and f**. The usual standard notation for the composite is $f \circ g$, which is read "f of g."

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Example **Composite Functions**

Given $f(x) = 2x - 3$ and $g(x) = 5x$, find $f \circ g$.

$$\begin{aligned}(f \circ g)(x) &= f(g(x)) \\ &= f(5x) \\ &= 2(5x) - 3 \\ &= 10x - 3\end{aligned}$$