What you'll learn about…

- **Radian Measure**
- **Graphs of Trigonometric Functions**
- **Periodicity**
- **Even and Odd Trigonometric Functions**
- **Transformations of Trigonometric Graphs**
- **Inverse Trigonometric Functions**

…and why

Trigonometric functions can be used to model periodic behavior and applications such as musical notes.

EQ:

What are trigonometric functions and how can we use them to solve applications?

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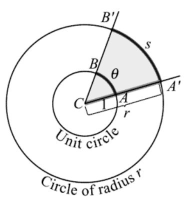
Radian Measure

- The radian measure of the angle *ACB* at the center of the unit circle equals the length of the arc that *ACB* cuts from the unit circle.



$$\theta = \frac{s}{r}$$

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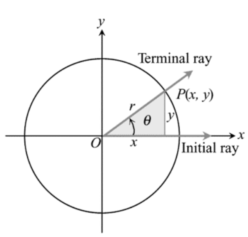
Radian Measure

- An angle of measure θ is placed in standard position at the center of circle of radius *r*,



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Trigonometric Functions of θ

The six basic trigonometric functions of θ are defined as follows:

sine: $\sin\theta = \frac{y}{r}$ cosecant: $\csc\theta = \frac{r}{y}$

cosine: $\cos\theta = \frac{x}{r}$ secant: $\sec\theta = \frac{r}{x}$

tangent: $\tan\theta = \frac{y}{x}$ cotangent: $\cot\theta = \frac{x}{y}$

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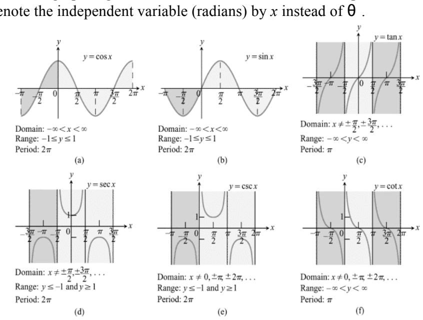
Graphs of Trigonometric Functions

- When we graph trigonometric functions in the coordinate plane, we usually denote the independent variable (radians) by *x* instead of θ .



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Angle Convention

Angle Convention: Use Radians

From now on in this book, it is assumed that all angles are measured in radians unless degrees or some other unit is stated explicitly. When we talk about the angle $\frac{\pi}{3}$ we mean $\frac{\pi}{3}$ radians (which is 60°), not $\frac{\pi}{3}$ degrees.

When you do calculus, keep your calculator in radian mode.

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Periodic Function, Period

A function $f(x)$ is periodic if there is a positive number p such that $f(x+p)=f(x)$ for every value of x. The smallest value of p is the period of f.

The functions $\cos x$, $\sin x$, $\csc x$ and $\sec x$ are periodic with period 2π.
The functions $\tan x$ and $\cot x$ are periodic with period π.

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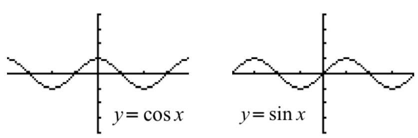
Even and Odd Trigonometric Functions

- The graphs of cos *x* and sec *x* are **even** functions because their graphs are symmetric about the *y*-axis.
- The graphs of sin *x*, csc *x*, tan *x* and cot *x* are **odd** functions.



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Example **Even and Odd Trigonometric Functions**

Show that csc *x* is an odd function.

$$\csc(-x)=\frac{1}{\sin(-x)}=\frac{1}{-\sin x}=-\csc x$$

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Transformations of Trigonometric Graphs

- The rules for shifting, stretching, shrinking and reflecting the graph of a function apply to the trigonometric functions.

Vertical stretch or shrink
Reflection about *x*-axis

Vertical shift

$$y = a\left[f\left(b\left(x + c\right)\right)\right] + d$$

Horizontal stretch or shrink
Reflection about the *y*-axis

Horizontal shift

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Example **Transformations of Trigonometric Graphs**

Determine the period, domain, range and draw the graph of

$$y = -2\sin\left(4x + \pi\right)$$

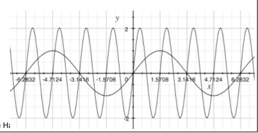
We can rewrite the function as $y = -2\sin\left(4\left(x + \frac{\pi}{4}\right)\right)$

The period of $y = a\sin bx$ is $\frac{2\pi}{b}$. In our example $b = 4$,

so the period is $\frac{2\pi}{4} = \frac{\pi}{2}$. The domain is $\left(-\infty, \infty\right)$.

The graph is a basic sin *x* curve with an amplitude of 2.
Thus, the range is [- 2, 2].

The graph of the function is shown together with the graph of the sin *x* function.



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Inverse Trigonometric Functions

- None of the six basic trigonometric functions graphed in Figure 1.42 is one-to-one. These functions do not have inverses. However, in each case, the domain can be restricted to produce a new function that does have an inverse.
- The domains and ranges of the inverse trigonometric functions become part of their definitions.

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Inverse Trigonometric Functions

Function	Domain	Range
$y = \cos^{-1} x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \sin^{-1} x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \tan^{-1} x$	$-\infty \le x \le \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$
$y = \sec^{-1} x$	$\lvert x \rvert \ge 1$	$0 \le y \le \pi$
$y = \csc^{-1} x$	$\lvert x \rvert \ge 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}, y \ne 0$
$y = \cot^{-1} x$	$-\infty \le x \le \infty$	$0 \le y \le \pi$

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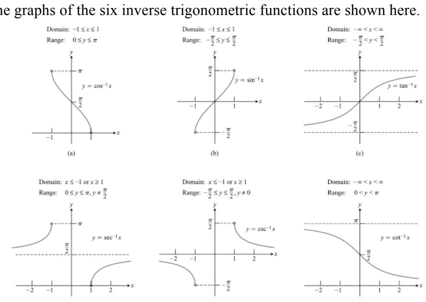
Inverse Trigonometric Functions

- The graphs of the six inverse trigonometric functions are shown here.



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Example **Inverse Trigonometric Functions**

Find the measure of $\sin^{-1}\left(-\frac{1}{2}\right)$ in degrees and in radians.

Put the calculator in degree mode and enter $\sin^{-1}\left(-\frac{1}{2}\right)$

The calculator returns $-30°$.

Put the calculator in radian mode and enter $\sin^{-1}\left(-\frac{1}{2}\right)$

The calculator returns $-.52359877556$ radians.

This is the same as $-\frac{\pi}{6}$ radians.

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