What you'll learn about…

- One-to-One Functions
- Inverses
- Finding Inverses
- Logarithmic Functions
- Properties of Logarithms
- Applications

…and why

Logarithmic functions are used in many applications including finding time in investment problems.

EQ:
What are logarithmic functions and how can we use them to solve applications?

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One-to-One Functions

- A function is a rule that assigns a single value in its range to each point in its domain.
- Some functions assign the same output to more than one input.
- Other functions never output a given value more than once.
- If each output value of a function is associated with exactly one input value, the function is *one-to-one*.

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One-to-One Functions

A function $f(x)$ is one-to-one on a domain D if $f(a) \neq f(b)$ whenever $a \neq b$.

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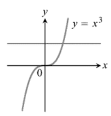
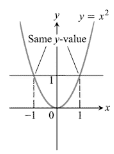
One-to-One Functions

The horizontal line test states that the graph of a one-to-one function $y = f(x)$ can intersect any horizontal line at most once.

If it intersects such a line more than once it assumes the same y-value more than once and is **not** a one-to-one function.



One-to-one: Graph meets each horizontal line once.



Not one-to-one: Graph meets some horizontal lines more than once.

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Inverses

- Since each output of a one-to-one function comes from just one input, a one-to-one function can be reversed to send outputs back to the inputs from which they came.
- The function defined by reversing a one-to-one function f is the **inverse of *f***.
- Composing a function with its inverse in either order sends each output back to the input from which it came.

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Inverses

The symbol for the inverse of f is f^{-1}, read "f inverse."

The -1 in f^{-1} is not an exponent; $f^{-1}(x)$ does not mean $\frac{1}{f(x)}$.

If $(f \circ g)(x) = (g \circ f)(x)$, then f and g are inverses of one another; otherwise they are not.

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Identity Function

The result of composing a function and its inverse in either order is the **identity function**.

The **identity function** is the function $f(x) = x$.

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Example **Inverses**

Determine via composition if $f(x) = \sqrt{x}$ and $g(x) = x^2$, $x > 0$, are inverses.

$(f \circ g)(x) = f(x^2) = \sqrt{x^2} = |x| = x$ (since $x > 0$)

$(g \circ f)(x) = g(\sqrt{x}) = (\sqrt{x})^2 = x$

Since $(f \circ g)(x) = (g \circ f)(x) = x$, the functions f and g ARE inverses of each other.

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Writing f^{-1} as a Function of x.

Solve the equation $y = f(x)$ for x in terms of y.

Interchange x and y. The resulting formula will be $y = f^{-1}(x)$.

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Writing f^{-1} as a Function of x.

Example: Find the inverse of $y = f(x) = 3x - 5$.

1) Solve for x in terms of y: $y = 3x - 5$
$3x = y + 5$
$x = (y + 5)/3$

2) Interchange x and y: $y = (x + 5)/3$

So, the inverse is $f^{-1}(x) = (x + 5)/3$

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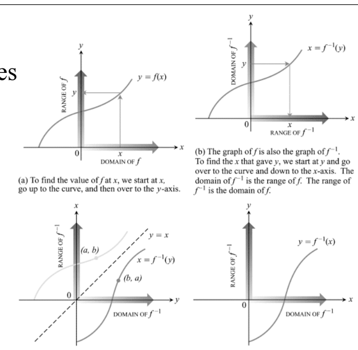
Finding Inverses



(a) To find the value of f at x, we start at x, go up to the curve, and then over to the y-axis.

(b) The graph of f is also the graph of f^{-1}. To find the x that gave y, we start at y and go over to the curve and down to the x-axis. The domain of f^{-1} is the range of f. The range of f^{-1} is the domain of f.

(c) To draw the graph of f^{-1} in the usual way, we reflect the system across the line $y = x$.

(d) Then we interchange the letters x and y. We now have a normal-looking graph of f^{-1} as a function of x.

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Example **Finding Inverses**

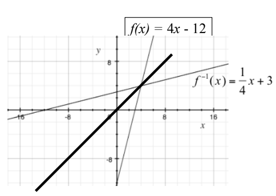
Given that $y = 4x - 12$ is one-to-one, find its inverse. Then graph the function and its inverse.

Solve the equation for x in terms of y.

$$x = \frac{y}{4} + 3$$

Interchange x and y.

$$y = \frac{x}{4} + 3$$



Notice the symmetry about the line $y = x$.

[-10,10] by [-15, 8]

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Base *a* Logarithmic Function

The base a logarithm function $y = \log_a x$ is the inverse of the base a exponential function $x = a^y$ ($a > 0$, $a \neq 1$).

The domain of $\log_a x$ is $(0, \infty)$, which is the range of a^x.
The range of $\log_a x$ is $(-\infty, \infty)$, which is the domain of a^x.

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Logarithmic Functions

- Logarithms with base e and base 10 are so important in applications that calculators have special keys for them.
- They also have their own special notations and names.

$y = \log_e x = \ln x$ is called the **natural logarithm function**.

$y = \log_{10} x = \log x$ is often called the **common logarithm function**.

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Inverse Properties for a^x and $\log_a x$

Base a: $a^{\log_a x} = x, \log_a a^x = x, a > 1, x > 0$

Base e: $e^{\ln x} = x, \ln e^x = x, x > 0$

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Properties of Logarithms

For any real numbers $x > 0$ and $y > 0$,

Product Rule : $\log_a xy = \log_a x + \log_a y$

Quotient Rule : $\log_a \frac{x}{y} = \log_a x - \log_a y$

Power Rule : $\log_a x^y = y \log_a x$

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Example **Properties of Logarithms**

Solve the following for *x*.

$2^x = 12$

$2^x = 12$

$\ln 2^x = \ln 12$ Take logarithms of both sides

$x \ln 2 = \ln 12$ PowerRule

$x = \frac{\ln 12}{\ln 2} = \frac{2.302585}{.693147} = 3.32193$

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Example **Properties of Logarithms**

Solve the following for *x*.

$e^x + 5 = 60$

$e^x + 5 = 60$

$e^x = 55$ Subtract 5

$\ln e^x = \ln 55$ Take logarithm of both sides

$x = \ln 55 =$ 4.007333

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Change of Base Formula

$\log_a x = \frac{\ln x}{\ln a}$ $\log_a x = \frac{\log_b x}{\log_b a}$

This formula allows us to evaluate $\log_a x$ for any base $a > 0$, $a \neq 1$, and to obtain its graph using the natural logarithm function on our grapher.

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Change of Base Formula Proof

Let $y = \log_a x$

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Example Population Growth

The population P of a city is given by $P = 105{,}300e^{0.015t}$ where $t = 0$ represents 1990. According to this model, when will the population reach 150,000?