Calculus Integration by Substitution Worksheet SOLUTIONS

Evaluate the following by hand.

1) $\int 6x^2\left(2x^3+1\right)^5 dx$ $\quad u = 2x^3+1$, $du = 6x^2\,dx$ $\quad \int u^5 du = \frac{u^6}{6} + C = \frac{\left(2x^3+1\right)^6}{6} + C$

2) $\int \frac{6x^2-3}{\sqrt{2x^3-3x}} dx$ $\quad u = 2x^3-3x$, $du = \left(6x^2-3\right)dx$ $\quad \int u^{-1/2} du = 2u^{1/2} + C = 2\sqrt{2x^3-3x} + C$

3) $\int \left(\sin(\cos x)\right)\left(\sin x\right)dx$ $\quad u = \cos x$, $du = -\sin x\,dx$ $\quad -\int \sin u\,du = -\left(-\cos u\right) + C = \cos\left(\cos x\right) + C)$

4) $\int \frac{e^{(1+1/x)}}{x^2} dx$ $\quad u = 1+\frac{1}{x}$, $du = -\frac{1}{x^2}dx$ $\quad -\int e^u du = -e^u + C = -e^{1+1/x} + C$

5) $\int \frac{\cos x}{\sin^3 x} dx$ $\quad u = \sin x$, $du = \cos x\,dx$ $\quad \int \frac{1}{u^3} du = \frac{u^{-2}}{-2} + C = -\frac{1}{2\sin^2 x} + C$

6) $\int \frac{1}{x\sqrt{\ln x}} dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int \frac{1}{\sqrt{u}} du = 2\sqrt{u} + C = 2\sqrt{\ln x} + C$

7) $\int_0^{\ln 2} e^{3x} dx$ $\quad u = 3x$, $du = 3dx$ $\quad \frac{1}{3}\int_0^{3\ln 2} e^u du = \frac{1}{3}\left[e^u\right]_0^{3\ln 2} = \frac{1}{3}\left(e^{3\ln 2} - e^0\right) = \frac{1}{3}\left(8-1\right) = \frac{7}{3}$

8) $\int_e^{e^2} \frac{1}{x\left(\ln x\right)^2} dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\quad \int_1^2 \frac{1}{u^2} du = \left[\frac{-1}{u}\right]_1^2 = \frac{-1}{2} - \left(\frac{-1}{1}\right) = \frac{1}{2}$