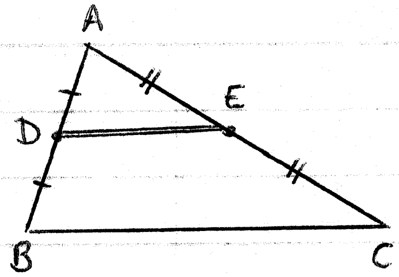
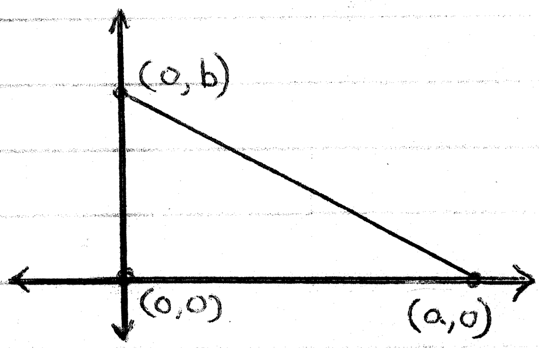
Geometry Ch 5-1 Exer, pg 300 #1-11, 12-18 (even), 21-26

1. In $\triangle ABC$, D is the midpoint of $\overline{AB}$, and E is the midpoint of $\overline{AC}$. $\overline{DE}$ is a <u>midsegment</u> of $\triangle ABC$.



2. Explain why it is convenient to place a right triangle as shown when writing a coord. proof. How might you want to re-label the vertices if the proof involves mid-points?

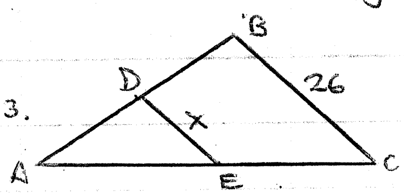


Try to get as many points on x- and y-axis so coordinate values are zero. This makes for easier calculations.

When working with midpoints proofs consider setting vertices at (0, 2b) and (2a, 0). This way your mid-point coordinates won't be fractions.

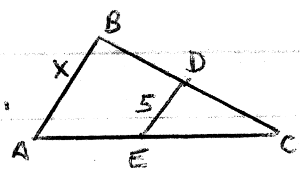
If $\overline{DE}$ is a midsegment of $\triangle ABC$, find the value of x.

3.



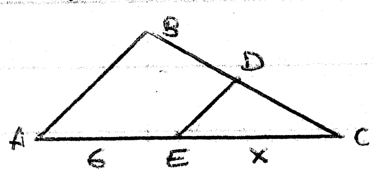
$x = 13$, half of $\overline{BC}$

4.



$x = 10$, twice midsegment $\overline{DE}$

5.

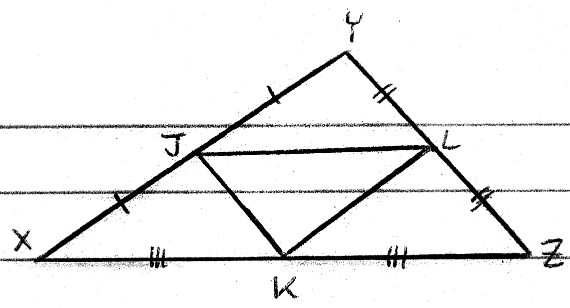


$x = 6$, E must be a midpt of $\overline{AC}$

In $\triangle XYZ$, $\overline{XJ} \cong \overline{JY}$,

$\overline{YL} \cong \overline{LZ}$,

$\overline{XK} \cong \overline{KZ}$



6. $\overline{JK} \parallel$ $\underline{\overline{YZ}}$ [Also $\overline{YL}$ and $\overline{LZ}$]

7. $\overline{JL} \parallel$ $\underline{\overline{XZ}}$ [also $\overline{XK}$ and $\overline{KX}$]

8. $\overline{XY} \parallel$ $\underline{\overline{KL}}$

9. $\overline{YJ} \cong$ $\underline{\overline{XJ} \text{ and } \overline{KL}}$

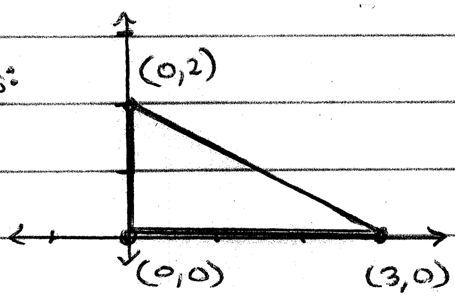
10. $\overline{JL} \cong$ $\underline{\overline{XK} \text{ and } \overline{KZ} \text{ and } \overline{XZ}}$

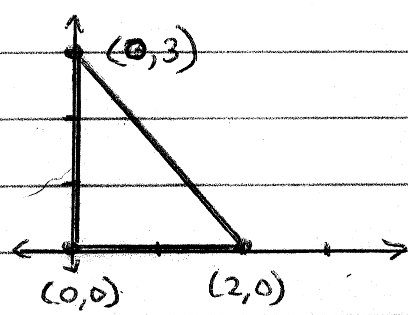
11. $\overline{JK} \cong$ $\underline{\overline{YL} \text{ and } \overline{LZ} \text{ and } \overline{YZ}}$

Place the figure in a coordinate plane. Assign coordinates to each vertex. [There are infinitely many correct answers! Try to choose a "convenient location.]

12. Right Triangle: leg lengths are 2 and 3.

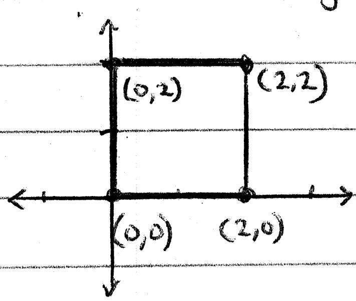
Two possibilities:

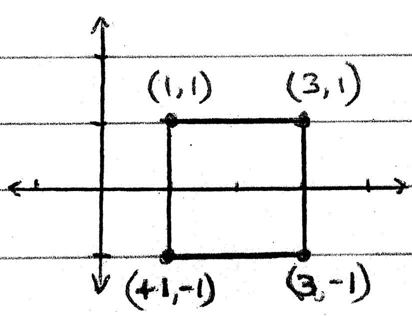




14. Square: side length is 2.

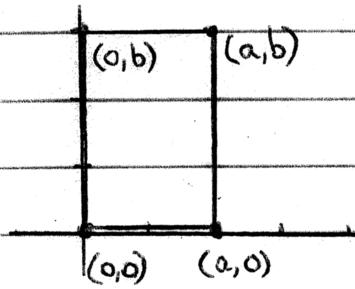
Two possibilities:

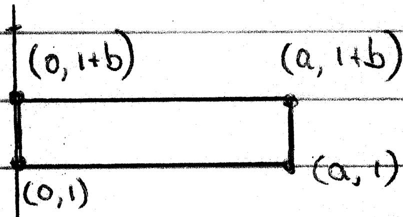




16. Rectangle: length is a, width is b.

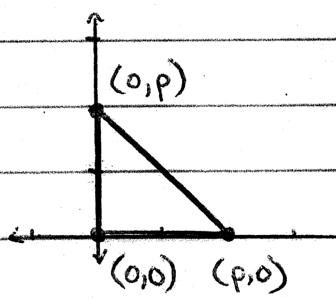
No way to know which is greater: a or b?

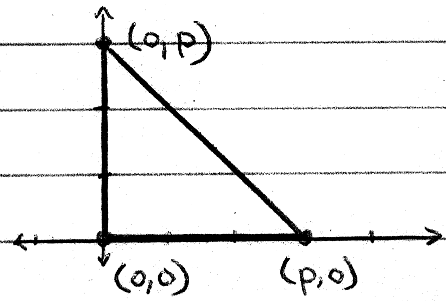




18. Isosceles right triangle: leg length is p.

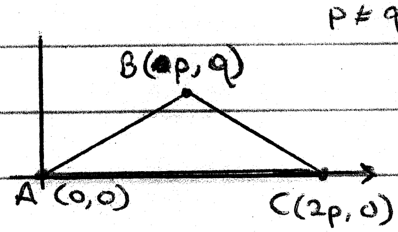
Two possibilities:





Sketch $\triangle ABC$. Find length / slope of each side; midpoint coordinates. Is $\triangle ABC$ right? Is $\triangle ABC$ isosceles? [Variables are positive, with $p \neq q$, $m \neq n$)]

21. $A(0,0)$, $B(p,q)$, $C(2p,0)$



Lengths:

$m\overline{AC} = 2p$

$m\overline{AB} = \sqrt{(p-0)^2 + (q-0)^2} = \sqrt{p^2+q^2}$

$m\overline{BC} = \sqrt{(2p-p)^2 + (0-q)^2} = \sqrt{p^2+q^2}$

Slopes

$AC: \frac{0-0}{2p-0} = \frac{0}{2p} = 0$

$AB: \frac{q-0}{p-0} = \frac{q}{p}$

$BC: \frac{0-q}{2p-p} = \frac{-q}{p}$

Midpoints:

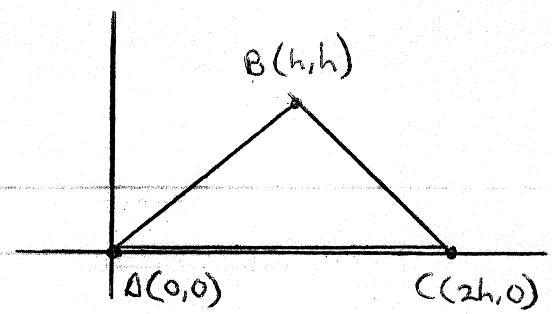
$AC: \left(\frac{0+2p}{2}, \frac{0+0}{2}\right) = (p, 0)$

$AB: \left(\frac{0+p}{2}, \frac{0+q}{2}\right) = \left(\frac{p}{2}, \frac{q}{2}\right)$

$BC: \left(\frac{p+2p}{2}, \frac{q+0}{2}\right) = \left(\frac{3p}{2}, \frac{q}{2}\right)$

Not Right; It is Isosceles

22. $A(0,0)$, $B(h,h)$, $C(2h,0)$



Lengths:

$AB: \sqrt{(h-0)^2+(h-0)^2} = \sqrt{2h^2} = h\sqrt{2}$

$BC: \sqrt{(2h-h)^2+(0-h)^2} = \sqrt{h^2+h^2} = \sqrt{2h^2} = h\sqrt{2}$

$AC: \ 2h$

Slopes:

$AB = \frac{h-0}{h-0} = \frac{h}{h} = 1$

$BC = \frac{h-0}{h-2h} = \frac{h}{-h} = -1$

$AC = \frac{0-0}{2h-0} = \frac{0}{2h} = 0$

Midpoints:

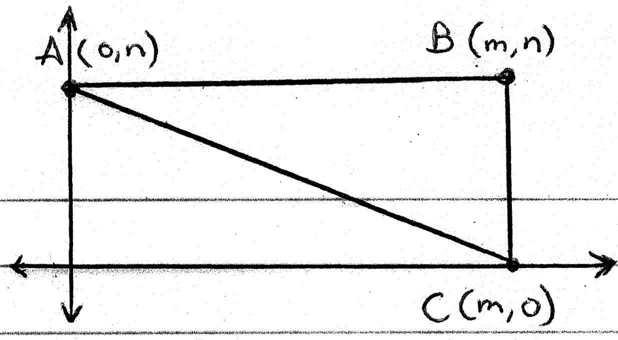
$AB: \left(\frac{0+h}{2}, \frac{0+h}{2}\right) = \left(\frac{h}{2}, \frac{h}{2}\right)$

$BC: \left(\frac{h+2h}{2}, \frac{h+0}{2}\right) = \left(\frac{3h}{2}, \frac{h}{2}\right)$

$AC: \left(\frac{0+2h}{2}, \frac{0+0}{2}\right) = (h, 0)$

$\triangle ABC$ is Right because slope of $\overline{AB}$ and $\overline{BC}$ are negative reciprocals

$\triangle ABC$ is Isosceles because the measure of $\overline{AB}$ is equal to $\overline{BC}$



23. $A(0,n)$, $B(m,n)$, $C(m,0)$

Lengths:

$AB = m$

$BC = n$

$AC = \sqrt{(0-m)^2+(n-0)^2} = \sqrt{m^2+n^2}$

Slopes:

$AB = \frac{n-n}{m-0} = \frac{0}{m} = 0$

$BC = \frac{n-0}{m-m} = \frac{n}{0} =$ undefined

$AC = \frac{n-0}{0-m} = -\frac{n}{m}$

Midpoints:

$AB: \left(\frac{0+m}{2}, \frac{n+n}{2}\right) = \left(\frac{m}{2}, \frac{2n}{2}\right) = \left(\frac{m}{2}, n\right)$

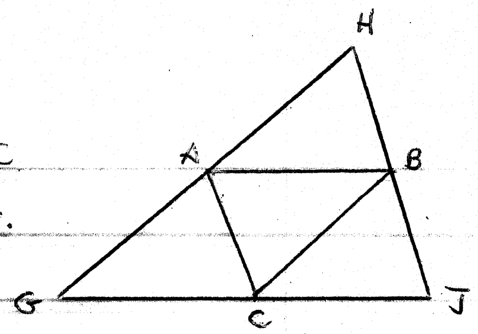
$BC: \left(\frac{m+m}{2}, \frac{n+0}{2}\right) = \left(\frac{2m}{2}, \frac{n}{2}\right) = \left(m, \frac{n}{2}\right)$

$AC: \left(\frac{0+m}{2}, \frac{n+0}{2}\right) = \left(\frac{m}{2}, \frac{n}{2}\right)$

ΔABC is right $\overline{AB}$ is a horizontal segment while $\overline{BC}$ is a vertical segment. There intersection is at a perpendiculer, creating a right angle.

ΔABC is not isosceles; no two sides are the same length.

ALGEBRA: Use $\triangle GHJ$, where A, B, and C are midpoints of the sides.



24. If $AB = 3x+8$ and $GJ = 2x+24$, what is AB?

$$2AB = GJ$$
$$2(3x+8) = 2x+24$$
$$6x+16 = 2x+24$$
$$4x = 8$$
$$x = 2$$

$$AB = 3(2)+8$$
$$= 6+8$$
$$\boxed{AB = 14}$$

25. If $AC = 3y-5$ and $HJ = 4y+2$, what is HB?

$$2AC = HJ$$
$$2(3y-5) = 4y+2$$
$$6y-10 = 4y+2$$
$$2y = 12$$
$$y = 6$$

$$HJ = 4(6)+2$$
$$HJ = 26$$
$$HB = \tfrac{1}{2}HJ = \tfrac{1}{2}(26)$$
$$\boxed{HB = 13}$$

26. If $GH = 7z-1$ and $BC = 4z-3$, what is GH?

$$GH = 2BC$$
$$7z-1 = 2(4z-3)$$
$$7z-1 = 8z-6$$
$$5 = z$$

$$GH = 7(5)-1$$
$$= 35-1$$
$$\boxed{GH = 34}$$