What you'll learn about…

- How Populations Grow
- Partial Fractions
- The Logistic Differential Equation
- Logistic Growth Model

… and why

Populations in the real world tend to grow logistically over extended periods of time.

EQ: What are partial fractions, and how do we apply them to a logistics growth model?

Example Antidifferentiating with a Partial Fraction Decomposition

So, how can we use this? Suppose we need to find $\int \frac{3x+6}{(x+5)(x-4)}dx$

$$\int \frac{3x+6}{(x+5)(x-4)}dx = \int \left(\frac{1}{x+5} + \frac{2}{x-4}\right)dx = \int \frac{dx}{x+5} + \int \frac{2}{x-4}dx$$

$$= \ln|x+5| + 2\ln|x-4| + C$$

$$= \ln\left|(x+5)(x-4)^2\right| + C$$

Example Finding a Partial Fraction Decomposition

$$f(x)=\frac{6x^2-8x-4}{(x^2-4)(x-1)} = \frac{6x^2-8x-4}{(x+2)(x-2)(x-1)} = \frac{A}{x+2}+\frac{B}{x-2}+\frac{C}{x-1}$$

$$6x^2-8x-4=A(x-2)(x-1)+B(x+2)(x-1)+C(x+2)(x-2)$$

When x = -2:
24 + 16 - 4 = 12A
36 = 12A
3 = A

When x = 2:
24 – 16 – 4 = 4B
4 = 4B
1 = B

When x = 1:
6 – 8 – 4 = -3C
-6 = -3C
2 = C

Example Antidifferentiating with Partial Fractions

Find $\int \frac{6x^2-8x-4}{(x-2)(x+2)(x-1)}dx$

$$= \int \left(\frac{1}{x-2} + \frac{3}{x+2} + \frac{2}{x-1} \right) dx$$

$$= \ln|x-2| + 3\ln|x+2| + 2\ln|x-1| + C$$

$$= \ln\left(|x-2||x+2|^3|x-1|^2 \right) + C$$

Assignment: p. 373 #5,6, 11-14

The General Logistic Formula

The solution of the general logistic differential equation

$$\frac{dP}{dt} = kP(M - P)$$

is

$$P = \frac{M}{1 + Ae^{-(Mk)t}}$$

where A is a constant determined by an appropriate initial condition. The **carrying capacity** M and the **growth constant** k are positive constants.