Calculus Review for Chapter 6 Solutions

1) $\int_0^{\pi/3} \sec^2\theta\, d\theta = \left[\tan\theta\right]_0^{\pi/3} = \tan\frac{\pi}{3} - \tan 0 = \sqrt{3} - 0 = \sqrt{3}$

2) $\int_1^2 \left(x + \frac{1}{x^2}\right) dx = \left[\frac{x^2}{2} - \frac{1}{x}\right]_1^2 = \left(2 - \frac{1}{2}\right) - \left(\frac{1}{2} - 1\right) = 2$

3)

$$\int_0^1 \frac{36dx}{(2x+1)^3} \quad u = 2x+1 \quad du = 2dx$$

$$36\left(\frac{1}{2}\right)\int_1^3 \frac{du}{u^3} = 18\left[-\frac{1}{2u^2}\right]_1^3 = 18\left[-\frac{1}{18} - \left(-\frac{1}{2}\right)\right] = -1 + 9 = 8$$

4)

$$\int_{-1}^1 2x\sin(1-x^2)\,dx \quad u = 1-x^2 \quad du = -2x\,dx$$

$$-\int_0^0 \sin u\, du = 0$$

5)

$$\int_0^{\pi/2} 5\sin^{3/2} x\cos x\, dx \quad u = \sin x \quad du = \cos x\, dx$$

$$5\int_0^1 u^{3/2}\, du = 5\left[\frac{u^{5/2}}{5/2}\right]_0^1 = 5\left(\frac{2}{5}(1) - \frac{2}{5}(0)\right) = 2$$

6) $\int_{1/2}^4 \frac{x^2 + 3x}{x} dx = \int_{1/2}^4 (x+3)\,dx = \left[\frac{x^2}{2} + 3x\right]_{1/2}^4 = (8+12) - \left(\frac{1}{8} + \frac{3}{2}\right) = 18\frac{3}{8}$

7)

$$\int_0^{\pi/4} e^{\tan x}\sec^2 x\, dx \quad u = \tan x \quad du = \sec^2 x\, dx$$

$$\int_0^1 e^u\, du = \left[e^u\right]_0^1 = \left(e^1 - e^0\right) = e - 1$$

8)

$$\int_1^e \frac{\sqrt{\ln r}}{r} dr \quad u = \ln r \quad du = \frac{1}{r} dr$$

$$\int_0^1 \sqrt{u}\, du = \left[\frac{u^{3/2}}{3/2}\right]_0^1 = \left(\frac{2}{3}(1) - \frac{2}{3}(0)\right) = \frac{2}{3}$$

9)

$$\int_0^1 \frac{x}{x^2 + 5x + 6} dx \quad \frac{x}{x^2 + 5x + 6} = \frac{A}{x+3} + \frac{B}{x+2} \Rightarrow x = A(x+2) + B(x+3)$$

$$x = -3: \quad -3 = -A \Rightarrow A = 3 \qquad x = -2: \quad -2 = B$$

$$\int_0^1 \frac{x}{x^2 + 5x + 6} dx = 3\int_0^1 \frac{dx}{x+3} - 2\int_0^1 \frac{dx}{x+2}$$

$$= 3\left[\ln|x+3|\right]_0^1 - 2\left[\ln|x+2|\right]_0^1 = 3(\ln 4 - \ln 3) - 2(\ln 3 - \ln 2) = 3\ln 4 - 5\ln 3 + 2\ln 2 = \ln\frac{4^3(2^2)}{3^5}$$

10) $\int_1^2 \frac{2x+6}{x^2-3x}\,dx \quad \frac{2x+6}{x^2-3x} = \frac{A}{x} + \frac{B}{x-3} \Rightarrow 2x+6 = A(x-3) + Bx$

$x = 3: \quad 12 = 3B \Rightarrow B = 4 \qquad x = 0: \quad 6 = -3A \Rightarrow A = -2$

$$\int_1^2 \frac{2x+6}{x^2-3x}\,dx = -2\int_1^2 \frac{dx}{x} + 4\int_1^2 \frac{dx}{x-3}\,dx$$

$$= -2\left[\ln|x|\right]_1^2 + 4\left[\ln|x-3|\right]_1^2 = -2(\ln 2 - \ln 1) + 4(\ln 1 - \ln 2) = -2\ln 2 - 4\ln 2 = \ln 2^{-6}$$

11) $\int \frac{\cos x}{2-\sin x}\,dx \quad u = 2 - \sin x \quad du = -\cos x\,dx$

$$-\int \frac{du}{u} = -\ln|u| + C = -\ln|2-\sin x| + C$$

12) $\int \frac{dx}{\sqrt[3]{3x+4}} \quad u = 3x+4 \quad du = 3\,dx$

$$\frac{1}{3}\int u^{-1/3}du = \frac{1}{3}\left(\frac{u^{2/3}}{2/3}\right) + C = \frac{1}{2}(3x+4)^{2/3} + C$$

13) $\int \frac{t\,dt}{t^2+5} \quad u = t^2+5 \quad du = 2t\,dt$

$$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|t^2+5| + C$$

14) $\int \frac{1}{\theta^2}\sec\frac{1}{\theta}\tan\frac{1}{\theta}\,d\theta \quad u = \frac{1}{\theta} \quad du = -\frac{1}{\theta^2}\,d\theta$

$$-\int \sec u \tan u\,du = -\sec u + C = -\sec\frac{1}{\theta} + C$$

15) $\int \frac{\tan(\ln y)}{y}\,dy \quad u = \ln y \quad du = \frac{1}{y}\,dy$

$$\int \tan u\,du = \ln|\sec u| + C = \ln|\sec(\ln y)| + C$$

16) $\int e^x \sec(e^x)\,dx \quad u = e^x \quad du = e^x\,dx$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C = \ln|\sec(e^x) + \tan(e^x)| + C$$

17) $\int \frac{dx}{x\ln x} \quad u = \ln x \quad du = \frac{1}{x}\,dx$

$$\int \frac{1}{u}\,du = \ln|u| + C = \ln|\ln x| + C$$

18) $\int \frac{dt}{t\sqrt{t}} = \int t^{-3/2}dt = \frac{t^{-1/2}}{-1/2} + C = -\frac{2}{\sqrt{t}} + C$

19) $\int x^3 \cos x \, dx$

x^3	$\cos x$
$3x^2$	$\sin x$
$6x$	$-\cos x$
6	$-\sin x$
0	$\cos x$

$\int x^3 \cos x \, dx = x^2 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x + C$

20) $\int x^4 \ln x \, dx$ $\quad u = \ln x \quad dv = x^4 dx$

$du = \frac{1}{x}dx \quad v = \frac{x^5}{5}$

$\int x^4 \ln x \, dx = x^5 \ln x - \frac{1}{5}\int x^4 \, dx + C = x^5 \ln x - \frac{x^5}{25} + C$

21) $\int e^{3x} \sin x \, dx$ $\quad u = e^{3x} \quad dv = \sin x dx$

$du = 3e^{3x}dx \quad v = -\cos x$

$\int e^{3x} \sin x \, dx = -e^{3x} \cos x - (-3)\int e^{3x} \cos x \, dx + C$ $\quad u = e^{3x} \quad dv = \cos x dx$

$du = 3e^{3x}dx \quad v = \sin x$

$\int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3\left(e^{3x} \sin x - 3\int e^{3x} \sin x \, dx\right) + C$

$\int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3e^{3x} \sin x - 9\int e^{3x} \sin x \, dx + C$

$10\int e^{3x} \sin x \, dx = -e^{3x} \cos x + 3e^{3x} \sin x + C$

$\int e^{3x} \sin x \, dx = \frac{e^{3x}}{10}\left(3\sin x - \cos x\right) + C$

22) $\int x^2 e^{-3x} \, dx$

x^2	e^{-3x}
$2x$	$-\frac{1}{3}e^{-3x}$
2	$\frac{1}{9}e^{-3x}$
0	$-\frac{1}{27}e^{-3x}$

$\int x^2 e^{-3x} \, dx = -\frac{1}{3}x^2 e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + C$

23) $\int \frac{25}{x^2-25}dx \qquad \frac{25}{x^2-25} = \frac{A}{x+5} + \frac{B}{x-5} \Rightarrow 25 = A(x-5) + B(x+5)$

$$x = 5 : 25 = 10B \Rightarrow B = \frac{5}{2} \qquad x = -5 : 25 = -10A \Rightarrow A = -\frac{5}{2}$$

$$\int \frac{25}{x^2-25}dx = -\frac{5}{2}\int \frac{dx}{x+5} + \frac{5}{2}\int \frac{dx}{x-5} = -\frac{5}{2}\ln|x+5| + \frac{5}{2}\ln|x-5| + C$$

24) $\int \frac{5x+2}{2x^2+x-1}dx \qquad \frac{5x+2}{2x^2+x-1} = \frac{A}{2x-1} + \frac{B}{x+1} \Rightarrow 5x+2 = A(x+1) + B(2x-1)$

$$x = -1 : -3 = -3B \Rightarrow B = 1 \qquad x = \frac{1}{2} : \frac{9}{2} = \frac{3}{2}A \Rightarrow A = 3$$

$$\int \frac{5x+2}{2x^2+x-1}dx = 3\int \frac{dx}{2x-1} + \int \frac{dx}{x+1} = 3\left(\frac{1}{2}\right)\ln|2x-1| + \ln|x+1| + C$$