Calculus page 395 ff

1)

$$\int_0^{\pi}\left(1-\cos^2 x\right)dx = \int_0^{\pi}\left(\sin^2 x\right)dx = \int_0^{\pi}\left(\frac{1}{2}-\frac{1}{2}\cos 2x\right)dx$$

$$= \left[\frac{x}{2}-\frac{1}{4}\sin 2x\right]_0^{\pi} = \left(\frac{\pi}{2}-\frac{1}{4}\sin 2\pi\right)-\left(\frac{0}{2}-\frac{1}{4}\sin 2(0)\right)$$

$$= \frac{\pi}{2}$$

2)

$$\int_{-\pi/3}^{\pi/3}\left(\frac{1}{2}\sec^2 t-\left(-4\sin^2 t\right)\right)dt = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\sec^2 t\,dt + 4\int_{-\pi/3}^{\pi/3}\left(\frac{1}{2}-\frac{1}{2}\cos 2t\right)dt$$

$$= \frac{1}{2}\left[\tan t\right]_{-\pi/3}^{\pi/3} + \left[2t\right]_{-\pi/3}^{\pi/3} - 4\left[\frac{1}{4}\sin 2t\right]_{-\pi/3}^{\pi/3}$$

$$= \frac{1}{2}\left(\tan\frac{\pi}{3}-\tan\left(-\frac{\pi}{3}\right)\right)+\left(\frac{2\pi}{3}-\left(\frac{-2\pi}{3}\right)\right)-\left(\sin\frac{2\pi}{3}-\sin\left(-\frac{2\pi}{3}\right)\right)$$

$$= \frac{1}{2}\left(\sqrt{3}-\left(-\sqrt{3}\right)\right)+\frac{4\pi}{3}-\left(\frac{\sqrt{3}}{2}-\left(-\frac{\sqrt{3}}{2}\right)\right) = \frac{4\pi}{3}$$

3)

$$\int_0^1\left(y^2-y^3\right)dy = \left[\frac{y^3}{3}-\frac{y^4}{4}\right]_0^1 = \left(\frac{1}{3}-\frac{1}{4}\right)-\left(0-0\right) = \frac{1}{12}$$

4)

$$\int_0^1\left(12y^2-12y^3-\left(2y^2-2y\right)\right)dy = \left[4y^3-3t^4-\frac{2y^3}{3}+y^2\right]_0^1$$

$$= \left(4-3-\frac{2}{3}+1\right)-\left(0-0-0+0\right) = \frac{4}{3}$$

5)

$$\int_{-2}^{2}\left(2x^2-\left(x^4-2x^2\right)\right)dx = \left[\frac{4x^3}{3}-\frac{x^5}{5}\right]_{-2}^{2}$$

$$= \left(\frac{32}{3}-\frac{32}{5}\right)-\left(-\frac{32}{3}-\left(-\frac{32}{5}\right)\right) = \frac{64}{3}-\frac{64}{5} = \frac{128}{15}$$

6)

$$\int_{-1}^{1}\left(x^2+2x^4\right)dx = \left[\frac{x^3}{3}+\frac{2x^5}{5}\right]_{-1}^{1}$$

$$= \left(\frac{1}{3}+\frac{2}{5}\right)-\left(-\frac{1}{3}+\left(-\frac{2}{5}\right)\right) = \frac{2}{3}+\frac{4}{5} = \frac{22}{15}$$

9)

$$\int_0^1\left(x-\frac{x^2}{4}\right)dx+\int_1^2\left(1-\frac{x^2}{4}\right)dx=\left[\frac{x^2}{2}-\frac{x^3}{12}\right]_0^1+\left[x-\frac{x^3}{12}\right]_1^2$$

$$=\left(\frac{1}{2}-\frac{1}{12}\right)-(0-0)+\left(2-\frac{8}{12}\right)-\left(1-\frac{1}{12}\right)=\frac{5}{12}+\frac{4}{3}-\frac{11}{12}=\frac{5}{6}$$

10)

$$\int_0^1 x^2dx+\int_1^2(2-x)\,dx=\left[\frac{x^3}{3}\right]_0^1+\left[2x-\frac{x^2}{2}\right]_1^2$$

$$=\left(\frac{1}{3}-0\right)+(4-2)-\left(2-\frac{1}{2}\right)=\frac{1}{3}+2-\frac{3}{2}=\frac{5}{6}$$

11)

$$y^2=x+1\Rightarrow x=y^2-1 \qquad y^2=3-x\Rightarrow x=3-y^2$$

$$y^2-1=3-y^2\Rightarrow 2y^2=4\Rightarrow y=\pm\sqrt{2}$$

$$\int_{-\sqrt{2}}^{\sqrt{2}}\left(\left(3-y^2\right)-\left(y^2-1\right)\right)dy=\left[4y-\frac{2y^3}{3}\right]_{-\sqrt{2}}^{\sqrt{2}}$$

$$=\left(4\sqrt{2}-\frac{4\sqrt{2}}{3}\right)-\left(-4\sqrt{2}-\left(-\frac{4\sqrt{2}}{3}\right)\right)=8\sqrt{2}-\frac{8\sqrt{2}}{3}\approx 7.542$$

12)

$$y^2=x+3\Rightarrow x=y^2-3 \qquad y=2x\Rightarrow x=\frac{1}{2}y$$

$$y^2-3=\frac{1}{2}y\Rightarrow 2y^2-y-6=0\Rightarrow (2y+3)(y-2)=0\Rightarrow y=2,-\frac{3}{2}$$

$$\int_{-3/2}^{2}\left(\frac{1}{2}y-\left(y^2-3\right)\right)dy=\left[\frac{y^2}{4}-\frac{y^3}{3}+3y\right]_{-3/2}^{2}$$

$$=\left(1-\frac{8}{3}+6\right)-\left(\frac{9}{16}-\left(-\frac{9}{8}\right)-\frac{9}{2}\right)=\frac{13}{3}-\left(-\frac{45}{16}\right)=\frac{343}{48}\approx 7.246$$

13)

$$\int_{-2}^{0}\left(2x^3-x^2-5x-\left(-x^2+3x\right)\right)dx+\int_0^2\left(-x^2+3x-\left(2x^3-x^2-5x\right)\right)dx$$

$$=\left[\frac{x^4}{2}-4x^2\right]_{-2}^{0}+\left[4x^2-\frac{x^4}{2}\right]_0^2$$

$$=(0-0)-(8-16)+(16-8)-(0-0)=16$$

14) $\int_{-2}^{-1}\left(-x+2-\left(4-x^2\right)\right)dx+\int_{-1}^{2}\left(4-x^2-\left(-x+2\right)\right)dx+\int_{2}^{3}\left(-x+2-\left(4-x^2\right)\right)dx$

$$=\left[-\frac{x^2}{2}-2x+\frac{x^3}{3}\right]_{-2}^{-1}+\left[2x-\frac{x^3}{3}+\frac{x^2}{2}\right]_{-1}^{2}+\left[-\frac{x^2}{2}-2x+\frac{x^3}{3}\right]_{2}^{3}$$

$$=\left(-\frac{1}{2}+2-\frac{1}{3}\right)-\left(-2+4-\frac{8}{3}\right)+\left(4-\frac{8}{3}+2\right)-\left(-2+\frac{1}{3}+\frac{1}{2}\right)+\left(-\frac{9}{2}-6+9\right)-\left(-2-4+\frac{8}{3}\right)$$

$$=\frac{7}{6}-\left(-\frac{2}{3}\right)+\frac{10}{3}-\left(-\frac{7}{6}\right)+\left(-\frac{3}{2}\right)-\left(-\frac{10}{3}\right)=\frac{14}{6}+\frac{22}{3}-\frac{3}{2}=\frac{49}{6}$$

15) $x^2-2=2\Rightarrow x^2=4\Rightarrow x=\pm2$

$$\int_{-2}^{2}\left(2-\left(x^2-2\right)\right)dx=\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$$

$$=\left(8-\frac{8}{3}\right)-\left(-8-\left(-\frac{8}{3}\right)\right)=\frac{16}{3}-\left(-\frac{16}{3}\right)=\frac{32}{3}$$

16) $2x-x^2=-3\Rightarrow x^2-2x-3=0\Rightarrow\left(x-3\right)\left(x+1\right)=0\Rightarrow x=3,-1$

$$\int_{-1}^{3}\left(2x-x^2-\left(-3\right)\right)dx=\left[x^2-\frac{x^3}{3}+3x\right]_{-1}^{3}$$

$$=\left(9-9+9\right)-\left(1-\left(-\frac{1}{3}\right)-3\right)=9-\left(-\frac{5}{3}\right)=\frac{32}{3}$$

17) $7-2x^2=x^2+4\Rightarrow 3x^2=3\Rightarrow x=\pm1$

$$\int_{-1}^{1}\left(7-2x^2-\left(x^2+4\right)\right)dx=\left[3x-x^3\right]_{-1}^{1}$$

$$=\left(3-1\right)-\left(-3-\left(-1\right)\right)=4$$

18)

$x^4-4x^2+4=x^2\Rightarrow x^4-5x^2+4=0\Rightarrow\left(x^2-4\right)\left(x^2-1\right)\Rightarrow x=\pm1,\pm2$

$$\int_{-2}^{-1}\left(x^2-\left(x^4-4x^2+4\right)\right)dx+\int_{-1}^{1}\left(x^4-4x^2+4-x^2\right)dx+\int_{1}^{2}\left(x^2-\left(x^4-4x^2+4\right)\right)dx$$

$$=\left[\frac{5x^3}{3}-\frac{x^5}{5}-4x\right]_{-2}^{-1}+\left[\frac{x^5}{5}-\frac{5x^3}{3}+4x\right]_{-1}^{1}+\left[\frac{5x^3}{3}-\frac{x^5}{5}-4x\right]_{1}^{2}$$

$$=\left(\frac{-5}{3}-\left(-\frac{1}{5}\right)+4\right)-\left(\frac{-40}{3}-\left(\frac{-32}{5}\right)+8\right)+\left(\frac{1}{5}-\frac{5}{3}+4\right)-\left(\frac{-1}{5}-\left(\frac{-5}{3}\right)-4\right)+\left(\frac{40}{3}-\frac{32}{5}-8\right)-\left(\frac{5}{3}-\frac{1}{5}-4\right)$$

$$=\frac{38}{15}-\left(\frac{16}{15}\right)+\frac{38}{15}-\left(-\frac{38}{15}\right)+\left(-\frac{16}{15}\right)-\left(-\frac{38}{15}\right)=\frac{120}{15}=8$$

19) $x\sqrt{a^2-x^2}=0 \Rightarrow x=0, a^2-x^2=0 \Rightarrow x=a$

$$\int_0^a x\sqrt{a^2-x^2}\,dx \quad u=a^2-x^2 \quad du=-2x\,dx$$

$$-\frac{1}{2}\int_{a^2}^0 \sqrt{u}\,du = -\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_{a^2}^0 = -\frac{1}{2}\left[\frac{2}{3}(0)-\frac{2}{3}a^3\right]=\frac{1}{3}a^3$$

#19 does not match the book answer…that is because the BOOK IS WRONG!

20) $\sqrt{|x|}=\frac{x+6}{5} \Rightarrow |x|=\frac{x^2+12x+36}{25}$

$x>0: 25x = x^2+12x+36 \Rightarrow x^2-13x+36=0 \Rightarrow x=4,9$

$x<0: -25x = x^2+12x+36 \Rightarrow x^2+37x+36=0 \Rightarrow x=-1,-36$

-36 is an extraneous result

21) $|x^2-4|=\frac{x^2}{2}+4$

$x^2-4>0: x^2-4=\frac{x^2}{2}+4 \Rightarrow 2x^2-8=x^2+8 \Rightarrow x^2=16 \Rightarrow x=4$

$x<0: -x^2+4=\frac{x^2}{2}+4 \Rightarrow -2x^2+8=x^2+8 \Rightarrow -3x^2=0 \Rightarrow x=0$

$$\int_0^4\left(\frac{x^2}{2}+4-\left(x^2-4\right)\right)dx=\left[-\frac{x^3}{6}+8x\right]_0^4=\left(-\frac{64}{6}+32\right)-(0+0)=\frac{128}{6}$$

22) $y^2=y+2 \Rightarrow y^2-y-2=0 \Rightarrow y=2,-1$

$$\int_{-1}^2\left(y+2-y^2\right)dy=\left[\frac{y^2}{2}+2y-\frac{y^3}{3}\right]_{-1}^2=\left(2+4-\frac{8}{3}\right)-\left(\frac{1}{2}-2+\frac{1}{3}\right)$$

$$=\frac{10}{3}-\left(-\frac{7}{6}\right)=\frac{27}{6}=\frac{9}{2}$$

23) $\frac{y^2}{4}-1=\frac{16+y}{4} \Rightarrow y^2-y-20=0 \Rightarrow y=5,-4$

$$\int_{-4}^5\left(4+\frac{y}{4}-\left(\frac{y^2}{4}-1\right)\right)dy=\left[5y+\frac{y^2}{8}-\frac{y^3}{12}\right]_{-4}^5=\left(25+\frac{25}{8}-\frac{125}{12}\right)-\left(-20+2-\left(-\frac{64}{12}\right)\right)$$

$$=\frac{425}{24}-\left(-\frac{38}{3}\right)=\frac{243}{8}=\frac{9}{2}$$

24) $y^2 = 3 - 2y^2 \Rightarrow 3y^2 = 3 \Rightarrow y = \pm 1$

$$\int_{-1}^{1} \left(3 - 2y^2 - y^2\right) dy = \left[3y - y^3\right]_{-1}^{1} = (3 - 1) - (-3 - (-1))$$
$$= 2 - (-2) = 4$$

25) $-y^2 = 2 - 3y^2 \Rightarrow 2y^2 = 2 \Rightarrow y = \pm 1$

$$\int_{-1}^{1} \left(2 - 3y^2 - (-y^2)\right) dy = \left[2y - \frac{2y^3}{3}\right]_{-1}^{1} = \left(2 - \frac{2}{3}\right) - \left(-2 - \left(-\frac{2}{3}\right)\right)$$
$$= \frac{4}{3} - \left(-\frac{4}{3}\right) = \frac{8}{3}$$

26) $4 - 4x^2 = x^4 - 1 \Rightarrow x^4 + 4x^2 - 5 = 0 \Rightarrow y = \pm 1$

$$\int_{-1}^{1} \left(4 - 4x^2 - (x^4 - 1)\right) dx = \left[5x - \frac{4x^3}{3} - \frac{x^5}{5}\right]_{-1}^{1} = \left(5 - \frac{4}{3} - \frac{1}{5}\right) - \left(-5 - \left(-\frac{4}{3}\right) - \left(-\frac{1}{5}\right)\right)$$
$$= \frac{52}{15} + \left(-\frac{52}{15}\right) = \frac{104}{15}$$

27) $3 - y^2 = -\frac{y^2}{4} \Rightarrow 12 - 4y^2 = -y^2 \Rightarrow y = \pm 2$

$$\int_{-2}^{2} \left(3 - y^2 - \left(-\frac{y^2}{4}\right)\right) dy = \left[3y - \frac{3y^3}{12}\right]_{-2}^{2} = \left(6 - \frac{24}{12}\right) - \left(-6 - \left(-\frac{24}{12}\right)\right)$$
$$= 4 - (-4) = 8$$

28) $2\sin x = \sin 2x \Rightarrow 2\sin x = 2\sin x \cos x \Rightarrow 2\sin x(1 - \cos x) = 0 \Rightarrow x = 0, \frac{\pi}{2}, \pi \text{ on } 0 \le x \le \pi$

$$\int_{0}^{\pi/2} (\sin 2x - 2\sin x)\, dx + \int_{\pi/2}^{\pi} (2\sin x - \sin 2x)\, dx = \int_{0}^{\pi/2} (2\sin x \cos x - 2\sin x)\, dx + \int_{\pi/2}^{\pi} (2\sin x - 2\sin x \cos x)\, d$$
$$= \int_{0}^{\pi/2} 2\sin x \cos x\, dx - \int_{0}^{\pi/2} 2\sin x\, dx + \int_{\pi/2}^{\pi} 2\sin x\, dx - \int_{\pi/2}^{2} 2\sin x \cos x\, dx$$
$$= 2\left[u^2\right]_{0}^{1} + \left[\cos x\right]_{0}^{\pi/2} + \left[-\cos x\right]_{\pi/2}^{\pi} - 2\left[u^2\right]_{1}^{0}$$
$$= 2(1 - 0) + (0 - 1) + (1 - 0) - 2(0 - 1) = 4$$

29) $8\cos x = \sec^2 x \Rightarrow 8\cos^3 x = 0 \Rightarrow \text{ nowhere on } -\frac{\pi}{3} \le x \le \frac{\pi}{3}$

$$\int_{-\pi/3}^{\pi/3} \left(8\cos x - \sec^2 x\right) dx = \int_{-\pi/3}^{\pi/3} (8\cos x)\, dx - \int_{-\pi/3}^{\pi/3} \left(\sec^2 x\right) dx$$
$$= 8\left[\sin x\right]_{-\pi/3}^{\pi/3} - \left[\tan x\right]_{-\pi/3}^{\pi/3}$$
$$= 8\left(\frac{\sqrt{3}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right) - \left(\sqrt{3} - \left(-\sqrt{3}\right)\right) = 8\sqrt{3} - \left(2\sqrt{3}\right) = 6\sqrt{3}$$

30) $\cos\left(\frac{\pi x}{2}\right) = 1 - x^2 \Rightarrow x = 0, \pm 1$ (graphing calculator used)

$$\int_{-1}^{0}\left(1 - x^2 - \cos\left(\frac{\pi x}{2}\right)\right)dx + \int_{0}^{1}\left(1 - x^2 - \cos\left(\frac{\pi x}{2}\right)\right)dx$$

$$= \left[x - \frac{x^3}{3} - \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)\right]_{-1}^{0} + \left[x - \frac{x^3}{3} - \frac{2}{\pi}\sin\left(\frac{\pi x}{2}\right)\right]_{0}^{1}$$

$$= (0 - 0 - 0) - \left(-1 - \left(-\frac{1}{3}\right) - \frac{2}{\pi}(-1)\right) + \left(1 - \frac{1}{3} - \frac{2}{\pi}\right) - (0 - 0 - 0)$$

$$= \frac{4}{3} - \frac{4}{\pi} \approx 0.0601$$

31) $\sin\left(\frac{\pi x}{2}\right) = x \Rightarrow x = 0, \pm 1$ (graphing calculator used)

$$\int_{-1}^{0}\left(x - \sin\left(\frac{\pi x}{2}\right)\right)dx + \int_{0}^{1}\left(\sin\left(\frac{\pi x}{2}\right) - x\right)dx$$

$$= \left[\frac{x^2}{2} - \left(-\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right)\right)\right]_{-1}^{0} + \left[-\frac{2}{\pi}\cos\left(\frac{\pi x}{2}\right) - \frac{x^2}{2}\right]_{0}^{1}$$

$$= \left(0 + \frac{2}{\pi}\right) - \left(\frac{1}{2} - 0\right) + \left(0 - \frac{1}{2}\right) - \left(-\frac{2}{\pi} - 0\right)$$

$$= \frac{4}{\pi} - 1 \approx 0.273$$

32) $$\int_{-\pi/4}^{\pi/4}\left(\sec^2 x - \tan^2 x\right)dx = \int_{-\pi/4}^{\pi/4}\left(\sec^2 x - \left(\sec^2 x - 1\right)\right)dx = \int_{-\pi/4}^{\pi/4} 1\,dx$$

$$= \left[x\right]_{-\pi/4}^{\pi/4} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

33) $\tan^2 y = -\tan^2 y \Rightarrow y = 0$

$$\int_{-\pi/4}^{0}\left(\tan^2 y - \left(-\tan^2 y\right)\right)dy + \int_{0}^{\pi/4}\left(\tan^2 y - \left(-\tan^2 y\right)\right)dy$$

$$= 2\int_{-\pi/4}^{0}\left(\sec^2 y - 1\right)dy + 2\int_{0}^{\pi/4}\left(\sec^2 y - 1\right)dy$$

$$= 2\left[\tan y - y\right]_{-\pi/4}^{0} + 2\left[\tan y - y\right]_{0}^{\pi/4}$$

$$= 2\left[(0 - 0) - \left(-1 - \left(-\frac{\pi}{4}\right)\right)\right] + 2\left[\left(1 - \frac{\pi}{4}\right) - (0 - 0)\right]$$

$$= 2\left(1 - \frac{\pi}{4}\right) + 2\left(1 - \frac{\pi}{4}\right) = 4 - \pi \approx 0.858$$

34) $3\sin y\sqrt{\cos y}=0 \Rightarrow y=0,\frac{\pi}{2}$ on $0\le y\le\frac{\pi}{2}$

$$\int_0^{\pi/2} 3\sin y\sqrt{\cos y}\,dy \qquad u=\cos y \qquad du=-\sin y\,dy$$

$$=-3\int_1^0 u^{1/2}\,du$$

$$=-3\left[\frac{2}{3}u^{3/2}\right]_1^0=-3\left(0-\frac{2}{3}\right)=2$$

37) $y^3=y\Rightarrow y^3-y=0\Rightarrow y=0,\pm1$

$$\int_{-1}^0\left(y^3-y\right)dy+\int_0^1\left(y-y^3\right)dy$$

$$=\left[\frac{y^4}{4}-\frac{y^2}{2}\right]_{-1}^0+\left[\frac{y^2}{2}-\frac{y^4}{4}\right]_0^1$$

$$=\left(0-0\right)-\left(\frac{1}{4}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)-\left(0-0\right)=\frac{1}{2}$$

38)

$$\frac{1}{x^2}=x\Rightarrow x^3=1\Rightarrow x=1$$

$$\int_0^1 x\,dx+\int_1^2\frac{1}{x^2}\,dx$$

$$=\left[\frac{x^2}{2}\right]_0^1+\left[\frac{x^{-1}}{-1}\right]_1^2$$

$$=\left(\frac{1}{2}-0\right)+\left(-\frac{1}{2}-\left(-1\right)\right)=\frac{1}{2}+\frac{1}{2}=1$$

39) $\sin x=\cos x\Rightarrow x=\frac{\pi}{4}$

$$\int_0^{\pi/4}\left(\cos x-\sin x\right)dx$$

$$=\left[\sin x+\cos x\right]_0^{\pi/4}$$

$$=\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}\right)-\left(0+1\right)=\sqrt{2}-1$$

40a) $3-x^2=-1\Rightarrow x^2=4\Rightarrow x=\pm2$

$$\int_{-2}^2\left(3-x^2-\left(-1\right)\right)dx$$

$$=\left[4x-\frac{x^3}{3}\right]_{-2}^2$$

$$=\left(8-\frac{8}{3}\right)-\left(-8-\left(-\frac{8}{3}\right)\right)=\frac{32}{3}$$

40b) $y = 3 - x^2 \Rightarrow x = \pm\sqrt{3-y}$

$$2\int_{-1}^{3}\sqrt{3-y}\,dy \qquad u = 3-y \quad du = -dy$$

$$= -2\left[\frac{u^{3/2}}{3/2}\right]_4^0 = -2\left[\frac{2}{3}u^{3/2}\right]_4^0$$

$$= -2\left(0 - \frac{16}{3}\right) = \frac{32}{3}$$

41a) $\left(-\sqrt{c}, c\right), \left(\sqrt{c}, c\right)$

41b) $\int_0^c 2\sqrt{y}\,dy = \int_c^4 2\sqrt{y}\,dy$

$$\left[\frac{4}{3}y^{3/2}\right]_0^c = \left[\frac{4}{3}y^{3/2}\right]_c^4$$

$$\frac{4}{3}c^{3/2} - 0 = \frac{4}{3}(4)^{3/2} - \frac{4}{3}c^{3/2}$$

$$\frac{8}{3}c^{3/2} = \frac{32}{3} \Rightarrow c = 4^{2/3}$$

41c) $\int_{-2}^{-\sqrt{c}}\left(4-x^2\right)dx + \int_{-\sqrt{c}}^{\sqrt{c}}(4-c)dx + \int_{\sqrt{c}}^{2}\left(4-x^2\right)dx = \int_{-\sqrt{c}}^{\sqrt{c}}\left(c-x^2\right)dx$

$$\left[4x - \frac{x^3}{3}\right]_{-2}^{-\sqrt{c}} + \left[4x - cx\right]_{-\sqrt{c}}^{\sqrt{c}} + \left[4x - \frac{x^3}{3}\right]_{\sqrt{c}}^{2} = \left[cx - \frac{x^3}{3}\right]_{-\sqrt{c}}^{\sqrt{c}}$$

$$\left[-4\sqrt{c} + \frac{c^{3/2}}{3} - \left(-8 + \frac{8}{3}\right)\right] + \left[4\sqrt{c} - c\sqrt{c} - \left(-4\sqrt{c} + c\sqrt{c}\right)\right] + \left[8 - \frac{8}{3} - \left(4\sqrt{c} - \frac{c^{3/2}}{3}\right)\right] = c\sqrt{c} - \frac{c^{3/2}}{3} - \left(-c\sqrt{c} + \frac{c^{3/2}}{3}\right)$$

$$\frac{2c^{3/2}}{3} + 16 - \frac{16}{3} - 2c^{3/2} = \frac{4c^{3/2}}{3} \Rightarrow \frac{8c^{3/2}}{3} = \frac{32}{3} \Rightarrow c^{3/2} = 4 \Rightarrow c = 4^{2/3}$$

42) $1 + \sqrt{x} = \frac{2}{\sqrt{x}} \Rightarrow \sqrt{x} + x = 2 \Rightarrow \sqrt{x} = 2 - x \Rightarrow x = 4 - 4x + x^2 \Rightarrow x = 1, 4$

4 is extraneous in this situation

$$\frac{x}{4} = \frac{2}{\sqrt{x}} \Rightarrow x^{3/2} = 8 \Rightarrow x = 8^{2/3} \Rightarrow x = 4$$

$$\int_0^1\left(1 + \sqrt{x} - \frac{x}{4}\right)dx + \int_1^4\left(\frac{2}{\sqrt{x}} - \frac{x}{4}\right)dx = \left[x + \frac{2}{3}x^{3/2} - \frac{x^2}{8}\right]_0^1 + \left[4\sqrt{x} - \frac{x^2}{8}\right]_1^4$$

$$= \left[1 + \frac{2}{3} - \frac{1}{8} - (0 + 0 - 0)\right] + \left[8 - 2 - \left(4 - \frac{1}{8}\right)\right] = \frac{37}{24} + \frac{17}{8} = \frac{88}{24} = \frac{11}{3}$$