Calculus Page 369 #5-8, 11-17 "uncontracted" solutions.

5) $3\ln|x| - 2\ln|x-4| + C$

6) $4\ln|x-2| - 2\ln|x+3| + C$

7) $x^2 + 4\ln|x+2| + 4\ln|x-2| + C$

8) $x + \frac{1}{2}\ln|x-3| - \frac{1}{2}\ln|x+3| + C$

11) $\ln|x-3| - \ln|2x+1| + C$

12) $\ln|3x+2| - 2\ln|x-1| + C$

13) $3\ln|x+1| + \ln|2x-3| + C$

14) $2\ln|x| + 3\ln|x+7| + C$

15) $3\ln|x| - \ln|x-2| + C$

16) $\ln|x-1| - \ln|x+1| + C$

17) $\ln|x+1| + \ln|x-1| - 2\ln|x| + C$

Solutions

1)

$$x - 12 = A(x-4) + Bx$$
$$Let\ x = 4: -8 = 4B \Rightarrow B = -2$$
$$Let\ x = 0: -12 = -4A \Rightarrow A = 3$$

2)

$$2x + 16 = A(x-2) + B(x+3)$$
$$Let\ x = 2: 20 = 5B \Rightarrow B = 4$$
$$Let\ x = -3: 10 = -5A \Rightarrow A = -2$$

3)

$$16 - x = A(x+5) + B(x-2)$$
$$Let\ x = -5: 21 = -7B \Rightarrow B = -3$$
$$Let\ x = 2: 14 = 7A \Rightarrow A = 2$$

4)

$$3 = A(x+3) + B(x-3)$$
$$Let\ x = -3: 3 = -6B \Rightarrow B = -\frac{1}{2}$$
$$Let\ x = 3: 3 = 6A \Rightarrow A = \frac{1}{2}$$

5)

$$\int \frac{x-12}{x^2-4x}\,dx = 3\int \frac{dx}{x} - 2\int \frac{dx}{x-4} = 3\ln|x| - 2\ln|x-4| + C$$

6)

$$\int \frac{2x+16}{x^2+x-6}\,dx = -2\int \frac{dx}{x+3} + 4\int \frac{dx}{x-2} = -2\ln|x+3| + 4\ln|x-2| + C$$

7)

$$\int \frac{2x^3}{x^2-4}\,dx \qquad \frac{2x^3}{x^2-4} = 2x + \frac{8x}{x^2-4}$$

$$\int \frac{2x^3}{x^2-4}\,dx = \int 2x\,dx + \int \frac{8x}{x^2-4}\,dx$$

$$8x = A(x-2) + B(x+2) \qquad x = 2: 16 = 4B \Rightarrow B = 4 \qquad x = -2: -16 = -4A \Rightarrow A = 4$$

$$\int 2x\,dx + 4\int \frac{dx}{x+2} + 4\int \frac{dx}{x-2} = x^2 + 4\ln|x+2| + 4\ln|x-2| + C$$

8)

$$\int \frac{x^2-6}{x^2-9}\,dx \qquad \frac{x^2-6}{x^2-9} = 1 + \frac{3}{x^2-9}$$

$$\int \frac{x^2-6}{x^2-9}\,dx = \int 1\,dx + \int \frac{3}{x^2-9}\,dx$$

$$3 = A(x-3) + B(x+3) \qquad x = 3: 3 = 6B \Rightarrow B = \frac{1}{2} \qquad x = -3: 3 = -6A \Rightarrow A = -\frac{1}{2}$$

$$\int 1\,dx - \frac{1}{2}\int \frac{dx}{x+3} + \frac{1}{2}\int \frac{dx}{x-3} = x - \frac{1}{2}\ln|x+3| + \frac{1}{2}\ln|x-3| + C$$

11)

$$\int \frac{7}{2x^2-5x-3}\,dx \qquad \frac{7}{2x^2-5x-3} = \frac{A}{2x+1} + \frac{B}{x-3}$$

$$7 = A(x-3) + B(2x+1) \qquad x = 3: 7 = 7B \Rightarrow B = 1 \qquad x = -\frac{1}{2}: 7 = -\frac{7}{2}A \Rightarrow A = -1$$

$$\int \frac{7}{2x^2-5x-3}\,dx = -1\int \frac{dx}{2x+1} + 1\int \frac{dx}{x-3} = -1\ln|2x+1| + 1\ln|x-3| + C$$

12)

$$\int \frac{1-3x}{3x^2-5x+2}\,dx \qquad \frac{1-3x}{3x^2-5x+2} = \frac{A}{3x-2} + \frac{B}{x-1}$$

$$1-3x = A(x-1) + B(3x-2) \qquad x = 1: -2 = B \qquad x = \frac{2}{3}: -1 = -\frac{1}{3}A \Rightarrow A = 3$$

$$\int \frac{1-3x}{3x^2-5x+2}\,dx = 3\int \frac{dx}{3x-2} - 2\int \frac{dx}{x-1} = \ln|3x-2| - 2\ln|x-1| + C$$

13) $\int \frac{8x-7}{2x^2-x-3}dx \qquad \frac{8x-7}{2x^2-x-3} = \frac{A}{2x-3} + \frac{B}{x+1}$

$8x-7 = A(x+1) + B(2x-3) \qquad x=-1: -15 = -5B \Rightarrow B = 3 \qquad x = \frac{3}{2}: 5 = \frac{5}{2}A \Rightarrow A = 2$

$\int \frac{8x-7}{2x^2-x-3}dx = 2\int \frac{dx}{2x-3} + 3\int \frac{dx}{x+1} = \ln|2x-3| + 3\ln|x-1| + C$

14) $\int \frac{5x+14}{x^2+7x}dx \qquad \frac{5x+14}{x^2+7x} = \frac{A}{x} + \frac{B}{x+7}$

$5x+14 = A(x+7) + Bx \qquad x=-7: -21 = -7B \Rightarrow B = 3 \qquad x = 0: 14 = 7A \Rightarrow A = 2$

$\int \frac{5x+14}{x^2+7x}dx = 2\int \frac{dx}{x} + 3\int \frac{dx}{x+7} = 2\ln|x| + 3\ln|x+7| + C$

15) $\frac{dy}{dx} = \frac{2x-6}{x^2-2x} \Rightarrow dy = \frac{2x-6}{x^2-2x}dx \Rightarrow \int dy = \int \frac{2x-6}{x^2-2x}dx \qquad \frac{2x-6}{x^2-2x} = \frac{A}{x} + \frac{B}{x-2}$

$2x-6 = A(x-2) + Bx \qquad x=2: -2 = 2B \Rightarrow B = -1 \qquad x = 0: -6 = -2A \Rightarrow A = 3$

$\int \frac{2x-6}{x^2-2x}dx = 3\int \frac{dx}{x} - 1\int \frac{dx}{x-2} = 3\ln|x| - \ln|x-2| + C$

16) $\frac{du}{dx} = \frac{2}{x^2-1} \Rightarrow du = \frac{2}{x^2-1}dx \Rightarrow \int du = \int \frac{2}{x^2-1}dx \qquad \frac{2}{x^2-1} = \frac{A}{x+1} + \frac{B}{x-1}$

$2 = A(x-1) + B(x+1) \qquad x=1: 2 = 2B \Rightarrow B = 1 \qquad x = -1: 2 = -2A \Rightarrow A = -1$

$u = \int \frac{2}{x^2-1}dx = -1\int \frac{dx}{x+1} + 1\int \frac{dx}{x-1} = \ln|x-1| - \ln|x+1| + C$

17) $\frac{dF}{dx} = \frac{2}{x^3-x} \Rightarrow dF = \frac{2}{x^3-x}dx \Rightarrow \int dF = \int \frac{2}{x^3-x}dx \qquad \frac{2}{x^3-x} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x}$

$2 = Ax(x-1) + Bx(x+1) + C(x+1)(x-1) \quad x=1: 2 = 2B \Rightarrow B = 1 \quad x=-1: 2 = 2A \Rightarrow A = 1 \quad x = 0: 2 = -C \Rightarrow C = -2$

$F = \int \frac{2}{x^3-x}dx = 1\int \frac{dx}{x+1} + 1\int \frac{dx}{x-1} - 2\int \frac{dx}{x} = \ln|x+1| + \ln|x-1| - 2\ln|x| + C$

18) $\frac{dG}{dt} = \frac{2t^3}{t^3-t} = 2 + \frac{2}{t^2-1} \Rightarrow dG = \left(2 + \frac{2t}{t^3-t}\right)dt \Rightarrow \int dG = \int 2\,dt + \int \frac{2}{t^2-1}dx \qquad \frac{2}{t^2-1} = \frac{A}{t+1} + \frac{B}{t-1}$

$2 = A(t-1) + B(t+1) + \quad t=1: 2 = 2B \Rightarrow B = 1 \quad t=-1: 2 = -2A \Rightarrow A = -1$

$G = 2t + \int \frac{2}{t^2-1}dt = 2t - 1\int \frac{dt}{t+1} + 1\int \frac{dt}{t-1} = 2t - \ln|t+1| + \ln|t-1| + C$

19) $\int \frac{2x}{x^2-4}\,dx \qquad \begin{array}{l} u = x^2 - 4 \\ du = 2x\,dx \end{array}$

$$\int \frac{du}{u} = \ln|u| + C = \ln\left|x^2 - 4\right| + C$$

20) $\int \frac{4x-3}{2x^2-3x+1}\,dx \qquad \begin{array}{l} u = 2x^2 - 3x + 1 \\ du = 4x - 3\,dx \end{array}$

$$\int \frac{du}{u} = \ln|u| + C = \ln\left|2x^2 - 3x + 1\right| + C$$

21) $\int \frac{x^2+x-1}{x^2-x}\,dx = \int \left(1 + \frac{2x-1}{x^2-x}\right)dx = \int 1\,dx + \int \frac{2x-1}{x^2-x}\,dx \qquad \begin{array}{l} u = x^2 - x \\ du = (2x-1)\,dx \end{array}$

$$x + \int \frac{du}{u} = x + \ln|u| + C = x + \ln\left|x^2 - x\right| + C$$

22) $\int \frac{2x^3}{x^2-1}\,dx = \int \left(2x + \frac{2x}{x^2-1}\right)dx = \int 2x dx + \int \frac{2x}{x^2-1}\,dx \qquad \begin{array}{l} u = x^2 - 1 \\ du = 2x\,dx \end{array}$

$$x^2 + \int \frac{du}{u} = x^2 + \ln|u| + C = x^2 + \ln\left|x^2 - 1\right| + C$$

23) a) 200 individuals b) 100 individuals c) $\frac{dP}{dt} = 0.006(100)(200-100) = 60$ individuals/yr

24) a) 700 individuals b) 350 individuals c) $\frac{dP}{dt} = 0.0008(350)(700-350) = 98$ individuals/yr

25) a) 1200 individuals b) 600 individuals c) $\frac{dP}{dt} = 0.0002(600)(1200-600) = 72$ individuals/yr

26) a) 5000 individuals b) 2500 individuals c) $\frac{dP}{dt} = 10^{-5}(2500)(5000-2500) = 62.5$ individuals/yr

Using my "cheating method" for finding the original logistic growth equation

27)

$$\frac{dP}{dt} = 0.006P(200 - P) \quad k = 0.006 \quad M = 200$$

$$P = \frac{200}{1 + Ae^{-1.2t}} \qquad 8 = \frac{200}{1 + A(1)} \Rightarrow 8 + 8A = 200 \Rightarrow A = 23$$

$$P = \frac{200}{1 + 24e^{-1.2t}}$$

28) $\frac{dP}{dt} = 0.0008P(700 - P) \quad k = 0.0008 \quad M = 700$

$P = \frac{700}{1 + Ae^{-.56t}} \qquad 10 = \frac{700}{1 + A(1)} \Rightarrow 10 + 10A = 700 \Rightarrow A = 69$

$P = \frac{700}{1 + 69e^{-.56t}}$

29) $\frac{dP}{dt} = 0.0002P(1200 - P) \quad k = 0.0002 \quad M = 1200$

$P = \frac{1200}{1 + Ae^{-.24t}} \qquad 20 = \frac{1200}{1 + A(1)} \Rightarrow 20 + 20A = 1200 \Rightarrow A = 59$

$P = \frac{1200}{1 + 59e^{-.24t}}$

30) $\frac{dP}{dt} = 10^{-5}P(5000 - P) \quad k = 0.00001 \quad M = 5000$

$P = \frac{5000}{1 + Ae^{-.05t}} \qquad 50 = \frac{5000}{1 + A(1)} \Rightarrow 50 + 50A = 5000 \Rightarrow A = 99$

$P = \frac{5000}{1 + 99e^{-.05t}}$

33a) $\frac{dP}{dt} = 0.0015P(150 - P) \quad k = 0.0015 \quad M = 150$

$P = \frac{150}{1 + Ae^{-.225t}} \qquad 6 = \frac{150}{1 + A(1)} \Rightarrow 6 + 6A = 150 \Rightarrow A = 24$

$P = \frac{150}{1 + 24e^{-.225t}}$

33b) $P = \frac{150}{1 + 24e^{-.225t}} \qquad 100 = \frac{150}{1 + 24e^{-.225t}} \Rightarrow 100 + 2400e^{-.225t} = 150$

$\Rightarrow e^{-.225t} = \frac{50}{2400} \Rightarrow t = \frac{\ln\left(\frac{1}{48}\right)}{-.225} \approx 17.21 \text{ weeks}$

$P = \frac{150}{1 + 24e^{-.225t}} \qquad 125 = \frac{150}{1 + 24e^{-.225t}} \Rightarrow 125 + 3000e^{-.225t} = 150$

$\Rightarrow e^{-.225t} = \frac{25}{3000} \Rightarrow t = \frac{\ln\left(\frac{1}{120}\right)}{-.225} \approx 21.28 \text{ weeks}$

34a) $\frac{dP}{dt} = 0.0004P\left(250 - P\right) \quad k = 0.0004 \quad M = 250$

$$P = \frac{250}{1 + Ae^{-.1t}} \qquad 28 = \frac{250}{1 + A\left(1\right)} \Rightarrow 28 + 28A = 250 \Rightarrow A = \frac{111}{14}$$

$$P = \frac{250}{1 + \frac{111}{14}e^{-.1t}}$$

34b) $P = \dfrac{250}{1 + \frac{111}{14}e^{-.1t}} \qquad 250 = \dfrac{250}{1 + \frac{111}{14}e^{-.1t}} \Rightarrow 250 + \dfrac{13875}{7}e^{-.1t} = 250$

$\Rightarrow e^{-.225t} = 0,$ which is impossible!

So, we must use a mathematical trick:

$$P = \frac{250}{1 + \frac{111}{14}e^{-.1t}} \qquad 249.5 = \frac{250}{1 + \frac{111}{14}e^{-.1t}} \Rightarrow 249.5 + 1978.179e^{-.1t} = 250$$

$$\Rightarrow 1978.179e^{-.1t} = 0.5 \Rightarrow e^{-.1t} = 0.0002527577 \Rightarrow t \approx \frac{\ln 0.0002527577}{-.1} = 82.83 \text{ yrs}$$

35)