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1)

$$\int x\sin x\,dx \qquad \begin{array}{ll} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{array}$$

$$-x\cos x - \int -\cos x\,dx = -x\cos x + \sin x + C$$

2)

$$\int xe^x\,dx \qquad \begin{array}{ll} u = x & dv = e^x\,dx \\ du = dx & v = e^x \end{array}$$

$$xe^x - \int e^x dx = xe^x - e^x + C$$

3)

$$\int 3te^{2t}\,dt \qquad \begin{array}{ll} u = 3t & dv = e^{2t}\,dt \\ du = 3dt & v = \frac{1}{2}e^{2t} \end{array}$$

$$\frac{3}{2}te^{2t} - \frac{3}{2}\int e^{2t}dt = \frac{3}{2}te^{2t} - \frac{3}{2}\left(\frac{1}{2}e^{2t}\right) + C = \frac{3}{2}te^{2t} - \frac{3}{4}e^{2t} + C$$

4)

$$\int 2t\cos(3t)\,dt \qquad \begin{array}{ll} u = 2t & dv = \cos(3t)\,dt \\ du = 2dt & v = \frac{1}{3}\sin(3t) \end{array}$$

$$\frac{2}{3}t\sin(3t) - \frac{2}{3}\int \sin(3t)dt = \frac{2}{3}t\sin(3t) - \frac{2}{3}\left(-\frac{1}{3}\cos(3t)\right) + C$$

$$= \frac{2}{3}t\sin(3t) + \frac{2}{9}\cos(3t) + C$$

5)

$$\int x^2\cos x\,dx \qquad \begin{array}{ll} u = x^2 & dv = \cos x\,dx \\ du = 2x\,dx & v = \sin x \end{array}$$

$$= x^2\sin x - 2\int x\sin x\,dx + C \qquad \begin{array}{ll} u = x & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{array}$$

$$= x^2\sin x - 2\left(-x\cos x - \int -\cos x\,dx\right) + C$$

$$= x^2\sin x + 2x\cos x - 2\sin x + C$$

6)

$$\int x^2e^{-x}\,dx \qquad \begin{array}{ll} u = x^2 & dv = e^{-x}\,dx \\ du = 2x\,dx & v = -e^{-x} \end{array}$$

$$= -x^2e^{-x} - \int -2xe^{-x}\,dx + C \qquad \begin{array}{ll} u = 2x & dv = e^{-x}\,dx \\ du = 2dx & v = -e^{-x} \end{array}$$

$$= -x^2e^{-x} - 2xe^{-x} - \int -2e^{-x}\,dx + C$$

$$= -x^2e^{-x} - 2xe^{-x} + 2\left(-e^{-x}\right) + C = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

7)

$$\int 3x^2e^{2x}\,dx \qquad \begin{array}{ll} u = 3x^2 & dv = e^{2x}\,dx \\ du = 6x\,dx & v = \frac{1}{2}e^{2x} \end{array}$$

$$= \frac{3}{2}x^2e^{2x} - \int 3xe^{2x}\,dx + C \qquad \begin{array}{ll} u = 3x & dv = e^{2x}\,dx \\ du = 3dx & v = \frac{1}{2}e^{2x} \end{array}$$

$$= \frac{3}{2}x^2e^{2x} - \left(\frac{3}{2}xe^{2x} - \int \frac{3}{2}e^{2x}\,dx\right) + C$$

$$= \frac{3}{2}x^2e^{2x} - \frac{3}{2}xe^{2x} + \frac{3}{4}e^{2x} + C$$

8)

$$\int x^2\cos\left(\frac{x}{2}\right)dx \qquad \begin{array}{ll} u = x^2 & dv = \cos\left(\frac{x}{2}\right)dx \\ du = 2x\,dx & v = 2\sin\left(\frac{x}{2}\right) \end{array}$$

$$= 2x^2\sin\left(\frac{x}{2}\right) - \int 4x\sin\left(\frac{x}{2}\right)dx + C \qquad \begin{array}{ll} u = 4x & dv = \sin\left(\frac{x}{2}\right)dx \\ du = 4dx & v = -2\cos\left(\frac{x}{2}\right) \end{array}$$

$$= 2x^2\sin\left(\frac{x}{2}\right) - \left(-8x\cos\left(\frac{x}{2}\right) - \int -8\cos\left(\frac{x}{2}\right)dx\right) + C$$

$$= 2x^2\sin\left(\frac{x}{2}\right) + 8x\cos\left(\frac{x}{2}\right) - 16\sin\left(\frac{x}{2}\right) + C$$

9)

$$\int y\ln y\,dy \qquad \begin{array}{ll} u = \ln y & dv = y\,dy \\ du = \frac{1}{y}dy & v = \frac{y^2}{2} \end{array}$$

$$= \frac{y^2}{2}\ln y - \int \frac{y}{2}dy + C = \frac{y^2}{2}\ln y - \frac{y^2}{4} + C$$

10)

$$\int t^2\ln t\,dt \qquad \begin{array}{ll} u = \ln t & dv = t^2\,dt \\ du = \frac{1}{t}dt & v = \frac{t^3}{3} \end{array}$$

$$= \frac{t^3}{3}\ln t - \int \frac{t^2}{3}dt + C = \frac{t^3}{3}\ln t - \frac{t^3}{9} + C$$

11)

$$dy = (x+2)\sin x\,dx \Rightarrow \int dy = \int (x+2)\sin x\,dx \qquad \begin{array}{ll} u = x+2 & dv = \sin x\,dx \\ du = dx & v = -\cos x \end{array}$$

$$y = -\cos x(x+2) - \int -\cos x\,dx + C \Rightarrow y = -\cos x(x+2) + \sin x + C$$

$$2 = -\cos(0)(2) + \sin(0) + C \Rightarrow C = 4$$

$$y = -\cos x(x+2) + \sin x + 4$$

12) $dy = 2xe^{-x}\,dx \Rightarrow \int dy = \int 2xe^{-x}\,dx$ $\quad u = 2x \quad dv = e^{-x}\,dx$, $\quad du = 2dx \quad v = -e^{-x}$

$$y = -2xe^{-x} - \int -2e^{-x}\,dx + C \Rightarrow y = -2xe^{-x} - 2e^{-x} + C$$

$$3 = -2(0)e^{0} - 2e^{0} + C \Rightarrow C = 5$$

$$y = -2xe^{-x} - 2e^{-x} + 5$$

13) $du = x\sec^2 x\,dx \Rightarrow \int du = \int x\sec^2 x\,dx$ $\quad u = x \quad dv = \sec^2 x\,dx$, $\quad du = dx \quad v = \tan x$

$$u = x\tan x - \int \tan x\,dx + C \Rightarrow u = x\tan x - \ln|\sec x| + C$$

$$1 = 0 - 0 + C \Rightarrow C = 1$$

$$u = x\tan x - \ln|\sec x| + 1$$

14) $dz = x^3 \ln x\,dx \Rightarrow \int dz = \int x^3 \ln x\,dx$ $\quad u = \ln x \quad dv = x^3\,dx$, $\quad du = \frac{1}{x}dx \quad v = \frac{x^4}{4}$

$$z = \frac{x^4}{4}\ln x - \int \frac{x^3}{4}dx + C \Rightarrow z = \frac{x^4}{4}\ln x - \frac{x^4}{16} + C$$

$$5 = \frac{1}{4}(0) - \frac{1}{16} + C \Rightarrow C = \frac{81}{16}$$

$$z = \frac{x^4}{4}\ln x - \frac{x^4}{16} + \frac{81}{16}$$

15) $dy = x\sqrt{x-1}\,dx \Rightarrow \int dy = \int x\sqrt{x-1}\,dx$ $\quad u = x-1 \quad x = u+1$, $\quad du = dx$

$$y = \int (u+1)u^{1/2}\,du \Rightarrow y = \frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$$

$$2 = \frac{2}{5}(0)^{5/2} + \frac{2}{3}(0)^{3/2} + C \Rightarrow C = 2$$

$$y = \frac{2}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + 2$$

Note: This answer differs from the book, as I did not use Integration by Parts, but the simpler Substitution Method. The solutions are equivalent.

Book version:

$dy = x\sqrt{x-1}\,dx \Rightarrow \int dy = \int x\sqrt{x-1}\,dx$ $\quad u = x \quad dv = \sqrt{x-1}\,dx$, $\quad du = dx \quad v = \frac{2}{3}(x-1)^{3/2}$

$$y = \frac{2}{3}x(x-1)^{3/2} - \frac{2}{3}\int (x-1)^{3/2}\,dx \Rightarrow y = \frac{2}{3}x(x-1)^{3/2} - \frac{2}{3}\left(\frac{2}{5}(x-1)^{5/2}\right) + C = \frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + C$$

$$2 = \frac{2}{3}(1)(0)^{3/2} + \frac{2}{5}(0)^{5/2} + C \Rightarrow C = 2$$

$$y = \frac{2}{3}x(x-1)^{3/2} - \frac{4}{15}(x-1)^{5/2} + 2$$

16) $dy = 2x\sqrt{x+2}\,dx \Rightarrow \int dy = \int 2x\sqrt{x+2}\,dx \qquad \begin{matrix} u = x+2 & x = u-2 \\ du = dx & \end{matrix}$

$$y = 2\int (u-2)u^{1/2}\,du \Rightarrow y = 2\left(\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2}\right) + C = \frac{4}{5}(x+2)^{5/2} - \frac{8}{3}(x+2)^{3/2} + C$$

$$0 = \frac{4}{5}(1)^{5/2} - \frac{8}{3}(1)^{3/2} + C \Rightarrow C = \frac{28}{15}$$

$$y = \frac{4}{5}(x+2)^{5/2} - \frac{8}{3}(x+2)^{3/2} + \frac{28}{15}$$

Again, this is equivalent to (and easier than) the book's version.

17)

$$\int e^x \sin x\,dx \qquad \begin{matrix} u = \sin x & dv = e^x dx \\ du = \cos x\,dx & v = e^x \end{matrix}$$

$$\int e^x \sin x\,dx = e^x \sin x - \int e^x \cos x\,dx + C \qquad \begin{matrix} u = \cos x & dv = e^x dx \\ du = -\sin x\,dx & v = e^x \end{matrix}$$

$$\int e^x \sin x\,dx = e^x \sin x - \left(e^x \cos x - \int -e^x \sin x\,dx\right) + C = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx + C$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x + C$$

$$\int e^x \sin x\,dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

18)

$$\int e^{-x} \cos x\,dx \qquad \begin{matrix} u = \cos x & dv = e^{-x} dx \\ du = -\sin x\,dx & v = -e^{-x} \end{matrix}$$

$$\int e^{-x} \cos x\,dx = -e^{-x} \cos x - \int e^{-x} \sin x\,dx + C \qquad \begin{matrix} u = \sin x & dv = e^{-x} dx \\ du = \cos x\,dx & v = -e^{-x} \end{matrix}$$

$$\int e^{-x} \cos x\,dx = -e^{-x} \cos x - \left(-e^{-x} \sin x - \int -e^{-x} \cos x\,dx\right) + C = -e^{-x} \cos x + e^{-x} \sin x - \int e^{-x} \cos x\,dx + C$$

$$2\int e^{-x} \cos x\,dx = -e^{-x} \cos x + e^{-x} \sin x + C$$

$$\int e^{-x} \cos x\,dx = \frac{-e^{-x} \cos x + e^{-x} \sin x}{2} + C$$

19)

$$\int e^x \cos 2x\,dx \qquad \begin{matrix} u = \cos 2x & dv = e^x dx \\ du = -2\sin 2x\,dx & v = e^x \end{matrix}$$

$$\int e^x \cos 2x\,dx = e^x \cos 2x - (-2)\int e^x \sin 2x\,dx + C \qquad \begin{matrix} u = \sin 2x & dv = e^x dx \\ du = 2\cos 2x\,dx & v = e^x \end{matrix}$$

$$\int e^x \cos 2x\,dx = e^x \cos 2x + 2\left(e^x \sin 2x - 2\int e^x \cos 2x\,dx\right) + C = -2e^x \cos 2x + 2e^x \sin 2x - 4\int e^x \cos 2x\,dx + C$$

$$5\int e^x \cos 2x\,dx = e^x \cos 2x + 2e^x \sin 2x + C$$

$$\int e^{-x} \cos x\,dx = \frac{e^x \cos 2x + 2e^x \sin 2x}{5} + C$$

20) $\int e^{-x}\sin 2x\,dx$ $\quad u=\sin 2x \quad dv=e^{-x}dx$ $\quad du=2\cos 2x\,dx \quad v=-e^{-x}$

$$\int e^{-x}\sin 2x\,dx=-e^{-x}\sin 2x-(-2)\int e^{-x}\cos 2x\,dx+C \qquad u=\cos 2x \quad dv=e^{-x}dx \quad du=-2\sin 2x\,dx \quad v=-e^{-x}$$

$$\int e^{-x}\sin 2x\,dx=-e^{-x}\sin 2x+2\left(-e^{-x}\cos 2x-2\int e^{-x}\sin 2x\,dx\right)+C$$

$$\int e^{-x}\sin 2x\,dx=-e^{-x}\sin 2x-2e^{-x}\cos 2x-4\int e^{-x}\sin 2x\,dx+C$$

$$5\int e^{-x}\sin 2x\,dx=-e^{-x}\sin 2x-2e^{-x}\cos 2x+C$$

$$\int e^{-x}\cos x\,dx=\frac{-e^{-x}\sin 2x-2e^{-x}\cos 2x}{5}+C$$

21)

u	dv
x^4	e^{-x}
$4x^3$	$-e^{-x}$
$12x^2$	e^{-x}
$24x$	$-e^{-x}$
24	e^{-x}
0	$-e^{-x}$

$$-x^4e^{-x}-4x^3e^{-x}-12x^2e^{-x}-24xe^{-x}-24e^{-x}+C$$

22)

u	dv
x^2-5x	e^x
$2x-5$	e^x
2	e^x
0	e^x

$$\left(x^2-5x\right)e^x-\left(2x-5\right)e^x+2e^x+C$$
$$=\left(x^2-7x+7\right)e^x+C$$

23)

u	dv
x^3	e^{-2x}
$3x^2$	$-\frac{1}{2}e^{-2x}$
$6x$	$\frac{1}{4}e^{-2x}$
6	$-\frac{1}{8}e^{-2x}$
0	$\frac{1}{16}e^{-2x}$

$$-\frac{1}{2}x^3e^{-2x}-\frac{3}{4}x^2e^{-2x}-\frac{3}{4}xe^{-2x}-\frac{3}{8}e^{-2x}+C$$

24)

u	dv
x^3	$\cos 2x$
$3x^2$	$\frac{1}{2}\sin 2x$
$6x$	$-\frac{1}{4}\cos 2x$
6	$-\frac{1}{8}\sin 2x$
0	$\frac{1}{16}\cos 2x$

$$\frac{1}{2}x^3\sin 2x+\frac{3}{4}x^2\cos 2x-\frac{3}{4}x\sin 2x-\frac{3}{8}\cos 2x+C$$

25)

u	dv
x^2	$\sin 2x$
$2x$	$-\frac{1}{2}\cos 2x$
2	$-\frac{1}{4}\sin 2x$
0	$\frac{1}{8}\cos 2x$

$$\left[-\frac{1}{2}x^2\cos 2x+\frac{1}{2}x\sin 2x+\frac{1}{4}\cos 2x\right]_0^{\pi/2}$$

$$=\left(-\frac{1}{2}\left(\frac{\pi}{2}\right)^2\cos 2\left(\frac{\pi}{2}\right)+\frac{1}{2}\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right)+\frac{1}{4}\cos 2\left(\frac{\pi}{2}\right)\right)-\left(-\frac{1}{2}(0)^2\cos 2(0)+\frac{1}{2}(0)\sin 2(0)+\frac{1}{4}\cos 2(0)\right)$$

$$=\left(\frac{\pi^2}{8}+0-\frac{1}{4}\right)-\left(0+0+\frac{1}{4}\right)=\frac{\pi^2}{8}-\frac{1}{2}$$

26)

u	dv
x^3	$\cos 2x$
$3x^2$	$\frac{1}{2}\sin 2x$
$6x$	$-\frac{1}{4}\cos 2x$
6	$-\frac{1}{8}\sin 2x$
0	$\frac{1}{16}\cos 2x$

26 cont’d

$$x^3 \sin 2x + \frac{3}{4}x^2 \cos 2x - \frac{3}{4}x \sin 2x - \frac{3}{8}\cos 2x \Big]_0^{\pi/2}$$

$$\left(\frac{1}{2}\left(\frac{\pi}{2}\right)^3 \sin 2\left(\frac{\pi}{2}\right) + \frac{3}{4}\left(\frac{\pi}{2}\right)^2 \cos 2\left(\frac{\pi}{2}\right) - \frac{3}{4}\left(\frac{\pi}{2}\right)\sin 2\left(\frac{\pi}{2}\right) - \frac{3}{8}\cos 2\left(\frac{\pi}{2}\right)\right) - \left(\frac{1}{2}(0)^3 \sin 2(0) + \frac{3}{4}(0)^2 \cos 2(0) - \frac{3}{4}(0)\sin 2(0) - \frac{3}{8}\cos 2(0$$

$$\left(0 + \frac{3\pi^2}{16}(-1) - 0 - \frac{3}{8}(-1)\right) - \left(0 + 0 - 0 - \frac{3}{8}(-1)\right) = -\frac{3\pi^2}{16} + \frac{3}{4}$$