Calculus Chapter 7 Part 2 Review SOLUTIONS

Use Separation of Variables to solve the initial value problem:

1) $\frac{dy}{dx} = \frac{2-x^2}{y+3}$ and $y = 1$ when $x = 2$

$$\int (y+3)\,dy = \int (2-x^2)\,dx$$

$$\frac{y^2}{2} + 3y = 2x - \frac{x^3}{3} + C$$

$$\frac{1}{2} + 3 = 4 - \frac{8}{3} + C \Rightarrow C = \frac{13}{6}$$

$$\frac{y^2}{2} + 3y = 2x - \frac{x^3}{3} + \frac{13}{6}$$

2) $\frac{dy}{dx} = 2x^3 - 2x + 2$ and $y = 3$ when $x = 0$

$$\int dy = \int (2x^3 - 2x + 2)\,dx$$

$$y = \frac{x^4}{2} - x^2 + 2x + C$$

$$3 = C$$

$$y = \frac{x^4}{2} - x^2 + 2x + 3$$

3) Find the amount of time required for an investment to quadruple if the annual rate is 2.95% and interest is compounded continuously. Round your answer to the nearest hundredth of a year.

$$A = Pe^{rt}$$

$$4 = e^{0.0295t}$$

$$t = \frac{\ln 4}{0.0295} \approx 46.99 \text{ years}$$

4) The decay equation for a radioactive substance is known to by $y = y_0 e^{-0.029t}$, with t in days. About how long will it take for the amount of substance to decay to 65% of its original value?

$$0.65 = e^{-0.029t}$$

$$t = \frac{\ln 0.65}{-0.029} \approx 14.85 \text{ years}$$

5) A certain radioactive isotope has a half-life of approximately 2,200 years. How many years, to the nearest year, would be required for a given amount of the isotope to decay to 40% of its original amount?

$$0.40 = e^{-\frac{\ln 2}{2200}t}$$

$$t = \frac{\ln 0.4}{-\frac{\ln 2}{2200}} \approx 2{,}908 \text{ years}$$

6) A cup of tea with temperature 183°F is placed in a refrigerator with temperature 34°F. After 8 minutes, the temperature of the tea is 122.5°F. When will its temperature by 98.6°F? Round your answer to the nearest minute. $\left(\text{Newton's Law of Cooling: } T - T_S = \left(T_O - T_S\right)e^{-kt}\right)$

$$122.5 - 34 = (183 - 34)e^{-8k}$$

$$\frac{88.5}{149} = e^{-8k} \Rightarrow k = \frac{\ln\left(\frac{88.5}{149}\right)}{-8}$$

$$98.6 - 34 = 149e^{\frac{\ln\left(\frac{88.5}{149}\right)}{8}t}$$

$$\frac{64.6}{149} = e^{\frac{\ln\left(\frac{88.5}{149}\right)}{8}t} \Rightarrow t = \frac{\ln\left(\frac{64.6}{149}\right)}{\frac{\ln\left(\frac{88.5}{149}\right)}{8}} \approx 12.83$$

About 13 minutes after it was put into the refrigerator.

7) The logistic differential equation $\frac{dP}{dt} = 0.001P(450 - P)$ describes the growth of a population P, where t is measured in years.

A) What is the carrying capacity of the population? 450

B) What is the size of the population when it is growing the fastest? 225