Calculus Chapter 7 Review Solutions

1) $$\int_0^5 \left(t^2 - 0.2t^3\right)dt = \left[\frac{t^3}{3} - \frac{t^4}{20}\right]_0^5 = \left(\frac{125}{3} - \frac{625}{20}\right) - (0-0) = \frac{2500-1875}{60} = \frac{625}{60} = 10\frac{5}{12} \text{ ft}$$

2) $$\int_0^7 \left(4 + 0.001t^4\right)dt = \left[4t + \frac{t^5}{5000}\right]_0^7 = \left(28 + \frac{16807}{5000}\right) - (0-0) \approx 31.361 \text{ gal.}$$

3) $$\int_0^{100} \left(21 - e^{0.03x}\right)dx = \left[21x - \frac{e^{0.03x}}{0.03}\right]_0^{100} = \left(2100 - \frac{100}{3}e^3\right) - \left(0 - \frac{100}{3}\right) = \frac{6400}{3} - \frac{100e^3}{3} \approx 1464 \text{ billboards}$$

4) $$\int_0^2 (11 - 4x)\,dx = \left[11x - 2x^2\right]_0^2 = (22-8) - (0-0) = 14 \text{ g}$$

5) $$\int_0^{24} \left(600 - 300\cos\left(\frac{\pi t}{12}\right)\right)dx = \left[600x - \frac{3600}{\pi}\sin\left(\frac{\pi t}{12}\right)\right]_0^{24} = (14400 - 0) - (0-0) = 14{,}400 \text{ kWh}$$

6) $$\int_1^2 \left(x - \frac{1}{x^2}\right)dx = \left[\frac{x^2}{2} + \frac{1}{x}\right]_1^2 = \left(2 + \frac{1}{2}\right) - \left(\frac{1}{2} - 1\right) = 1$$

7) $$x + 1 = 3 - x^2 \Rightarrow x^2 + x - 2 = 0 \Rightarrow (x+2)(x-1) = 0 \Rightarrow x = -2, 1$$

$$\int_{-2}^1 \left(3 - x^2 - (x+1)\right)dx = \left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-2}^1 = \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - 2 + \frac{8}{3}\right) = \frac{9}{2}$$

8) $$\sqrt{x} + \sqrt{y} = 1 \Rightarrow \sqrt{y} = 1 - \sqrt{x} \Rightarrow \left(\sqrt{y}\right)^2 = \left(1 - \sqrt{x}\right)^2 \Rightarrow y = 1 - 2\sqrt{x} + x$$

$$\int_0^1 \left(1 - 2\sqrt{x} + x\right)dx = \left[x - 2\left(\frac{x^{3/2}}{3/2}\right) + \frac{x^2}{2}\right]_0^1 = \left(1 - \frac{4}{3} + \frac{1}{2}\right) - (0 - 0 - 0) = \frac{1}{6}$$

9) $$\int_0^3 2y^2\,dy = \left[\frac{2y^3}{3}\right]_0^3 = (18 - 0) = 18$$

10) $$y^2 - 4 = y + 16 \Rightarrow y^2 - y - 20 = 0 \Rightarrow (y-5)(y+4) = 0 \Rightarrow y = -4, 5$$

$$\frac{1}{4}\int_{-4}^5 \left(y + 16 - \left(y^2 - 4\right)\right)dy = \frac{1}{4}\left[20y + \frac{y^2}{2} - \frac{y^3}{3}\right]_{-4}^5 = \frac{1}{4}\left[\left(100 + \frac{25}{2} - \frac{125}{3}\right) - \left(-80 + 8 + \frac{64}{3}\right)\right] = 30.375$$

11) $$\int_0^{\pi/4} (x - \sin x)\,dx = \left[\frac{x^2}{2} + \cos x\right]_0^{\pi/4} = \left(\frac{\pi^2}{32} + \frac{\sqrt{2}}{2}\right) - (0 + 1) \approx 0.0155$$

12) $$\int_0^{\pi}(2\sin x-\sin 2x)\,dx=\left[-2\cos x+\frac{1}{2}\cos 2x\right]_0^{\pi}=\left(2+\frac{1}{2}\right)-\left(-2+\frac{1}{2}\right)=4$$

13) Solved with a graphing calculator:

$$\cos x=4-x^2\Rightarrow x\approx-2.128,2.128$$
$$\int_{-2.128}^{2.128}\left(4-x^2-\cos x\right)dx\approx 8.902$$

14) Solved with a graphing calculator:

$$\sec^2 x=3-|x|\Rightarrow x\approx-0.826,0.826$$
$$\int_{-0.826}^{0.826}\left(3-|x|-\sec^2 x\right)dx\approx 2.104$$

15)
$$1+\cos x=2-\cos x\Rightarrow 2\cos x=1\Rightarrow \cos x=\frac{1}{2}\Rightarrow y=-\frac{\pi}{3},\frac{\pi}{3}$$
$$\int_{-\pi/3}^{\pi/3}\left(1+\cos x-(2-\cos x)\right)dx=\left[-x+2\sin x\right]_{-\pi/3}^{\pi/3}=\left(-\frac{\pi}{3}+2\left(\frac{\sqrt{3}}{2}\right)\right)-\left(-\left(-\frac{\pi}{3}\right)+2\left(-\frac{\sqrt{3}}{2}\right)\right)=2\sqrt{3}-\frac{2\pi}{3}$$

16) $$\int_{\pi/3}^{5\pi/3}\left(2-\cos x-(1+\cos x)\right)dx=\left[x-2\sin x\right]_{\pi/3}^{5\pi/3}=\left(\frac{5\pi}{3}-2\left(-\frac{\sqrt{3}}{2}\right)\right)-\left(\frac{\pi}{3}-2\left(\frac{\sqrt{3}}{2}\right)\right)=\frac{4\pi}{3}+2\sqrt{3}$$

17) Solved with a graphing calculator:

$$x^3-x=\frac{x}{x^2+1}\Rightarrow x\approx-1.189,0,1.189$$
$$\int_{-1.189}^{0}\left(x^3-x-\frac{x}{x^2+1}\right)dx+\int_0^{1.189}\left(\frac{x}{x^2+1}-\left(x^3-x\right)\right)dx\approx 1.296$$

18) Solved with a graphing calculator:

$$3^{1-x^2}=\frac{x^2-3}{10}\Rightarrow x\approx-1.893,1.893$$
$$\int_{-1.893}^{1.893}\left(3^{1-x^2}-\frac{x^2-3}{10}\right)dx\approx 5.731$$

19) $$\int_{-\pi}^{\pi}\left(x\sin x-(-x\sin x)\right)dx=2\int_{-\pi}^{\pi}x\sin x\,dx \qquad \begin{array}{ll} u=x & dv=\sin x\,dx \\ du=dx & v=-\cos x\end{array}$$
$$2\int_{-\pi}^{\pi}x\sin x\,dx=2\left\{\left[-x\cos x\right]_{-\pi}^{\pi}-\int_{-\pi}^{\pi}-\cos x\,dx\right\}=2\left\{\left[-x\cos x\right]_{-\pi}^{\pi}+\left[\sin x\right]_{-\pi}^{\pi}\right\}$$
$$=2\left\{\left(-\pi\cos\pi-\left(-(-\pi)\cos(-\pi)\right)\right)+\sin\pi-\sin(-\pi)\right\}=2\{\pi+\pi+0-0\}=4\pi$$

20) $$\int_{-1}^{1} \pi\left(3x^4\right)^2 dx = \pi\int_{-1}^{1} 9x^8\, dx = 9\pi\left[\frac{x^9}{9}\right]_{-1}^{1} = 9\pi\left[\frac{1}{9} - \left(-\frac{1}{9}\right)\right] = 2\pi$$

21a) $$\pi\int_0^4 \left(\left(2\sqrt{x}\right)^2 - x^2\right)dx = \pi\int_0^4 \left(4x - x^2\right)dx = \pi\left[2x^2 - \frac{x^3}{3}\right]_0^4 = \pi\left[\left(32 - \frac{64}{3}\right) - (0-0)\right] = \frac{32}{3}\pi$$

21b) $$\pi\int_0^4 \left(y^2 - \left(\frac{y^2}{4}\right)^2\right)dy = \pi\int_0^4 \left(y^2 - \frac{y^4}{16}\right)dy = \pi\left[\frac{y^3}{3} - \frac{y^5}{80}\right]_0^4 = \pi\left[\left(\frac{64}{3} - \frac{1024}{80}\right) - (0-0)\right] = \frac{128}{15}\pi$$

21c) $$\pi\int_0^4 \left(\left(4 - \frac{y^2}{4}\right)^2 - (4-y)^2\right)dy = \pi\int_0^4 \left(16 - 2y^2 + \frac{y^4}{16} - \left(16 - 8y + y^2\right)\right)dy = \pi\int_0^4 \left(8y - 3y^2 + \frac{y^4}{16}\right)dy$$

$$= \pi\left[4y^2 - y^3 + \frac{y^5}{80}\right]_0^4 = \pi\left[\left(64 - 64 + \frac{1024}{80}\right) - (0-0)\right] = \frac{64}{5}\pi$$

21d) $$\pi\int_0^4 \left((4-x)^2 - \left(4 - 2\sqrt{x}\right)^2\right)dx = \pi\int_0^4 \left(16 - 8x + x^2 - \left(16 - 16\sqrt{x} + 4x\right)\right)dx = \pi\int_0^4 \left(16\sqrt{x} - 12x + x^2\right)dx$$

$$= \pi\left[\frac{16x^{3/2}}{3/2} - 6x^2 + \frac{x^3}{3}\right]_0^4 = \pi\left[\left(\frac{256}{3} - 96 + \frac{64}{3}\right) - (0 - 0 + 0)\right] = \frac{32}{3}\pi$$

22a) $$\pi\int_0^2 \left(\left(\sqrt{2y}\right)^2\right)dy = \pi\int_0^2 (2y)\,dx = \pi\left[y^2\right]_0^2 = \pi[4-0] = 4\pi$$

22b) $$\pi\int_0^k \left(\left(\sqrt{2y}\right)^2\right)dy = \pi\int_0^k (2y)\,dx = \pi\left[y^2\right]_0^k = \pi\left[k^2 - 0\right] = k^2\pi$$

22c) $V = k^2\pi \qquad \dfrac{dV}{dt} = 2k\pi\dfrac{dk}{dt} \qquad 2 = 2(1)\pi\dfrac{dk}{dt} \Rightarrow \dfrac{dk}{dt} = \dfrac{1}{\pi}$ units per sec.

23) $$\frac{4x^2}{121} + \frac{y^2}{12} = 1 \Rightarrow 48x^2 + 121y^2 = 1452 \Rightarrow y = \sqrt{\frac{1452 - 48x^2}{121}}$$

$$V = \pi\int_{-11/2}^{11/2} \left(\sqrt{\frac{1452 - 48x^2}{121}}\right)^2 dx \approx 276 \text{ in}^3$$

24) $$\int_0^{\pi} \pi(\sin x)^2\, dx = \frac{\pi^2}{2}$$

25) $$\pi\int_0^{\ln 2} \left(\left(e^{x/2}\right)^2 - 1^2\right)dx = \pi\int_0^{\ln 3} \left(e^x - 1\right)dx = \pi\left[e^x - x\right]_0^{\ln 3} = \pi\left[\left(e^{\ln 3} - \ln 3\right) - \left(e^0 - 0\right)\right] = \pi(2 - \ln 3)$$

26)

$$\sqrt{4-x^2}=\sqrt{3}\Rightarrow x=1$$

$$2\pi\int_{-2}^{-1}\left(\sqrt{4-x^2}\right)^2 dx+\pi\left(\sqrt{3}\right)^2(2)=2\pi\int_{-2}^{-1}\left(4-x^2\right)dx+6\pi$$

$$=2\pi\left[4x-\frac{x^3}{3}\right]_{-2}^{-1}+6\pi=2\pi\left[\left(-4-\left(-\frac{1}{3}\right)\right)-\left(-8-\left(-\frac{8}{3}\right)\right)\right]+6\pi+2\pi\left[\frac{5}{3}\right]=\frac{28}{3}\pi$$

27) Set up by hand, use GC for antiderivative.

$$9-x^2=0\Rightarrow \pm 3$$

$$\int_{-3}^{3}\sqrt{1+(-2x)^2}\,dx=\int_{-3}^{3}\sqrt{1+4x^2}\,dx\approx 19.494$$

28) Set up by hand, use GC for antiderivative.

$$x^3-x=x-x^3\Rightarrow 2x^3-2x=0\Rightarrow x=0,\pm 1$$

$$\int_{-1}^{1}\sqrt{1+\left(3x^2-1\right)^2}\,dx+\int_{-1}^{1}\sqrt{1+\left(1-3x^2\right)^2}\,dx\approx 5.245$$

29) Set up by hand, use GC for antiderivative.

$$y'=3x^2-6x \qquad y'=0\Rightarrow 3x(x-2)=0\Rightarrow x=0,2$$

$$y''=6x-6 \qquad y''(0)=-6\Rightarrow \text{local max} \qquad y''(2)=6\Rightarrow \text{local min}$$

$$\int_{0}^{2}\sqrt{1+\left(3x^2-6x\right)^2}\,dx\approx 4.592 \qquad \frac{4.592\text{ units}}{2\text{ units/sec}}=2.296\text{ sec.}$$

30a)

$$y=k\sin x \qquad y'=k\cos x \qquad \int_{0}^{2\pi}\sqrt{1+(k\cos x)^2}\,dx=\int_{0}^{2\pi}\sqrt{1+k^2\cos^2 x}\,dx$$

$$y=\sin kx \qquad y'=\frac{1}{k}\cos kx \qquad \int_{0}^{2\pi}\sqrt{1+\left(\frac{1}{k}\cos kx\right)^2}\,dx=\int_{0}^{2\pi}\sqrt{1+\frac{\cos^2 kx}{k^2}}$$

#30a is not complete

31)

$$F'=\sqrt{x^4-1} \qquad \int_{2}^{5}\sqrt{1+\left(\sqrt{x^4-1}\right)^2}\,dx=\int_{2}^{5}\sqrt{x^4}\,dx=\int_{2}^{5}x^2\,dx$$

$$=\left[\frac{x^3}{3}\right]_{2}^{5}=\frac{125}{3}-\frac{8}{3}=\frac{117}{3}=39$$

32a)

$$W=\int_{0}^{40}(100)\,dx=\left[100x\right]_{0}^{40}=4000-0=4000\text{ J}$$

32b)

$$W=\int_{0}^{40}0.8(40-x)\,dx=\left[32x-0.4x^2\right]_{0}^{40}=\left(1280-0.4(1600)\right)-(0-0)=640\text{ J}$$

32c) 4000 J + 640 J = 4,640 J

33) $$W = \int_0^{4750} 8\left(800 - \frac{400}{4750}x\right)dx = 22{,}800{,}000 \text{ ft - lb}$$

34) $$80 = k(0.3) \Rightarrow k = \frac{800}{3} \qquad W = \int_0^{0.3} \frac{800}{3}x\,dx = \left[\frac{400}{3}x^2\right]_0^{0.3} = 12 - 0 = 12 \text{ J}$$

$$\int_{0.3}^{1.3} \frac{800}{3}x\,dx = \left[\frac{400}{3}x^2\right]_{0.3}^{1.3} = 225\frac{1}{3} - 12 = 213\frac{1}{3} \text{ J}$$

36) $$\int_{-8}^{-2} 0.04\pi\left(\sqrt{64 - y^2}\right)^2(-y)\,dy \approx 113.097 \text{ in - lb}$$

37) $$\int_0^2 80(4y)(2 - y)\,dy \approx 426.667 \text{ ft - lb}$$

38) Note: 57 lbs per cubic FOOT!

Base: $\frac{5.75}{12} \times \frac{3.5}{12} \times \frac{10}{12} \times 57 \approx 6.638 \text{ lb}$

5.75” wide side: $\int_0^{5/6} 57\left(\frac{5.75}{12}\right)\left(\frac{5}{6} - y\right)dy \approx 9.484 \text{ lb}$

3.5” wide side: $\int_0^{5/6} 57\left(\frac{3.5}{12}\right)\left(\frac{5}{6} - y\right)dy \approx 5.773 \text{ lb}$

39) $$x^{1/2} + y^{1/2} = \sqrt{6} \Rightarrow y = \left(\sqrt{6} - \sqrt{x}\right)^2 = 6 - 2\sqrt{6x} + x$$

$$\int_0^6 \left(6 - 2\sqrt{6x} + x\right)^2 dx = 14.4$$