Calculus Section 4.5 Linearization and Newton's Method

1) a) $L(x) = 7 + 10(x-2) = 10x - 13$

b) $f(2.1) = 8.061 \quad L(2.1) = 10(2.1) - 13 = 8$

$L(x)$ is accurate to 10^{-1}

2) a) $L(x) = 5 - \frac{4}{5}(x+4) = -\frac{4}{5}x + \frac{9}{5}$

b) $f(-3.9) = 4.9204 \quad L(-3.9) = -0.8(-3.9) + 1.8 = 4.92$

$L(x)$ is accurate to 10^{-3}

3) a) $L(x) = 2 + 0(x-1) = 2$

b) $f(1.1) = 2.00909 \quad L(1.1) = 2$

$L(x)$ is accurate to 10^{-2}

4) a) $L(x) = 0 + 1(x-0) = x$

b) $f(0.1) = 0.0953 \quad L(0.1) = 0.1$

$L(x)$ is accurate to 10^{-2}

5) a) $L(x) = 0 + 1(x-\pi) = x - \pi$

b) $f(\pi + 0.1) = 0.10033 \quad L(\pi + 0.1) = 0.1$

$L(x)$ is accurate to 10^{-3}

6) a) $L(x) = \frac{\pi}{2} - 1(x-0) = \frac{\pi}{2} - x$

b) $f(0.1) = 1.4706 \quad L(0.1) = 1.4708$

$L(x)$ is accurate to 10^{-3}

7) $f'(x) = k(1+x)^{k-1} \quad f'(0) = k(1+0)^{k-1} = 1 \quad L(x) = 1 + k(x-0) = 1 + kx$

8) a) $(1.002)^{100} = (1+0.002)^{100} \approx 1 + 100(0.002) = 1.2 \quad (1.002)^{100} \approx 1.221$

$L(x)$ is accurate to 10^{-1}

b) $\sqrt[3]{1.009} = (1+0.009)^{1/3} \approx 1 + \frac{1}{3}(0.009) = 1.003 \quad \sqrt[3]{1.009} \approx 1.00299$

$L(x)$ is accurate to 10^{-5}

9) a) $L(x) = 1 - 6x$

b) $\frac{2}{1-x} = 2(1-x)^{-1} \quad L(x) = 2(1 - 1(-x)) = 2 + 2x$

c) $\frac{1}{\sqrt{1+x}} = (1+x)^{-1/2} \quad L(x) = 1 - \frac{1}{2}x$

10) a) $(4+3x)^{1/3} = \left(4\left(1+\frac{3}{4}x\right)\right)^{1/3} = \sqrt[3]{4}\left(1+\frac{3}{4}x\right)^{1/3} \quad L(x) = \sqrt[3]{4}\left(1+\frac{1}{3}\left(\frac{3}{4}x\right)\right) = \sqrt[3]{4} + \frac{\sqrt[3]{4}}{4}x$

b) $\sqrt{2+x^2} = \left(2\left(1+\frac{x^2}{2}\right)\right)^{1/2} = \sqrt{2}\left(1+\frac{x^2}{2}\right)^{1/2} \quad L(x) = \sqrt{2}\left(1+\frac{1}{2}\left(\frac{x^2}{2}\right)\right) = \sqrt{2} + \frac{\sqrt{2}}{4}x^2$

c) $\sqrt[3]{\left(1-\frac{1}{2+x}\right)^2} = \left(1-\frac{1}{2+x}\right)^{2/3} \quad L(x) = 1+\frac{2}{3}\left(-\frac{1}{2+x}\right) = 1-\frac{2}{6+3x}$

11) $y=\sqrt{x}$ at $x=100 \quad L(x) = 10+\frac{1}{20}(x-100) \quad L(101) = 10+\frac{1}{20}(101-100) = 10.05$

12) $y=\sqrt[3]{x}$ at $x=27 \quad L(x) = 3+\frac{1}{27}(x-27) \quad L(26) = 3+\frac{1}{27}(26-27) = 2\frac{26}{27} \approx 2.963$

13) $y=\sqrt[3]{x}$ at $x=1000 \quad L(x) = 10+\frac{1}{1000}(x-1000) \quad L(998) = 10+\frac{1}{1000}(998-1000) = 9.998$

14) $y=\sqrt{x}$ at $x=81 \quad L(x) = 9+\frac{1}{18}(x-81) \quad L(80) = 9+\frac{1}{18}(80-81) = 8\frac{17}{18} \approx 8.944$

15)

x	$f(x) = x^3 + x - 1$	$f'(x) = 2x + 1$	$f(x) / f'(x)$
1	1	3	1/3
2/3	-0.037037037	7/3	-0.0158730159
0.68253969	0.000507904514	2.36507938	0.00021475157
0.68232494	-0.000006867526	2.36464988	-0.000002904246
0.68232784	0.0000000968714	2.36465568	0.0000000409664
0.68232780			

To six places of accuracy, the solution is 0.682328

16) There are two solutions

x	$f(x) = x^4 + x - 3$	$f'(x) = 4x^3 + 1$	$f(x) / f'(x)$
1	-1	5	-0.2
1.2	0.2736	7.912	0.034580384226
1.16541961577	0.010134681268	7.33150511808	0.001382346613
1.16403726916	0.000015559884	7.30900174544	0.000002138866
1.16403514029	0.000000000037	7.30896713059	0.000000000005
1.16403514029			

To six places of accuracy, the solution is 1.164035

The second solution follows on the next page!

16)

x	$f(x) = x^4 + x - 3$	$f'(x) = 4x^3 + 1$	$f(x) / f'(x)$
-1	-3	-3	1
-2	11	-31	-0.354838709677
-1.64516129032	2.68028231085	-16.8108824813	-0.159437335538
-1.48572395479	0.386783392937	-12.1182036395	-0.031917551846
-1.45380640294	0.013300122778	-11.2907878702	-0.001177962329
-1.45262844061	0.000017587034	-11.2609358406	-0.000001561774
-1.45262687884	0.000000000031	-11.2608962941	-0.000000000003
-1.45262687883			

To six places of accuracy, the solution is -1.452627

17) There are two solutions

x	$f(x) = x^2 - 2x + 1 - \sin x$	$f'(x) = 2x - 2 - \cos x$	$f(x) / f'(x)$
0.5	-0.2294235538604	-1.87758256189	0.122191984129
0.377808015871	0.018238903313	-2.17385942395	-0.08390102466
0.386198118336	0.000083468812	-2.15395156662	-0.00003875148
0.386236869816	0.000000001785	-2.153859467647	-0.000000000829
0.386236870645			

To six places of accuracy, the solution is 0.386237

Second solution:

x	$f(x) = x^2 - 2x + 1 - \sin x$	$f'(x) = 2x - 2 - \cos x$	$f(x) / f'(x)$
2	0.090702573174	2.41614683655	0.037540174216
1.96245982578	0.002053578862	2.30664612524	-0.000890287782
1.961569538	0.000001158953	2.30404252746	0.000000503009
1.96156903499			

To six places of accuracy, the solution is 1.961569

18) There are two solutions

x	$f(x) = x^4 - 2$	$f'(x) = 4x^3$	$f(x) / f'(x)$
1.2	0.0736	6.912	0.010648148148
1.18935185185	0.000973847352	6.72962788677	0.000144710431
1.18920714142	0.00000017772	6.72717177036	0.000000026418
1.189207115			

To six places of accuracy, the solution is 1.189207

Second solution: since the function is even, we know that the other solution must be -1.189207

19) a) $dy = \left(3x^2 - 3\right)dx$ b) $dy = \left(3(2)^2 - 3\right)\left(0.05\right) = 0.45$

20) a) $dy=\left(\dfrac{\left(1+x^2\right)(2)-2x(2x)}{\left(1+x^2\right)^2}\right)dx=\left(\dfrac{2-2x^2}{\left(1+x^2\right)^2}\right)dx$ b) $dy=\left(\dfrac{2-2(-2)^2}{\left(1+(-2)^2\right)^2}\right)(0.1)=\left(\dfrac{-6}{25}\right)(0.1)=-0.024$

21) a) $dy=\left(x+2x\ln x\right)dx$ b) $dy=\left(1+2(1)\ln(1)\right)(0.1)=0.1$

22) a) $dy=\left(\dfrac{-x^2}{\sqrt{1-x^2}}+\sqrt{1-x^2}\right)dx$ b) $dy=\left(\dfrac{-0^2}{\sqrt{1-0^2}}+\sqrt{1-0^2}\right)(-0.2)=-0.2$

23) a) $dy=\left(e^{\sin x}\left(\cos x\right)\right)dx$ b) $dy=\left(e^{\sin\pi}\left(\cos\pi\right)\right)(-0.1)=0.1$

24) a) $dy=\left(-3\csc\left(1-\dfrac{x}{3}\right)\cot\left(1-\dfrac{x}{3}\right)\left(-\dfrac{1}{3}\right)\right)dx=\left(\csc\left(1-\dfrac{x}{3}\right)\cot\left(1-\dfrac{x}{3}\right)\right)dx$

b) $dy=\left(\csc\left(\dfrac{2}{3}\right)\cot\left(\dfrac{2}{3}\right)\right)(0.1)\approx 0.206$

25) a) $y=\dfrac{x}{1+x}$ $dy=\dfrac{(1+x)(1)-x(1)}{(1+x)^2}dx=\left(\dfrac{1}{(1+x)^2}\right)dx$ b) $dy=\left(\dfrac{1}{(1+0)^2}\right)(0.01)=0.01$

26) a) $y=\dfrac{x^2}{2+x}$ $dy=\dfrac{(2+x)2x-x^2(1)}{(2+x)^2}dx=\left(\dfrac{4x+x^2}{(2+x)^2}\right)dx$ b) $dy=\left(\dfrac{4(2)+(2)^2}{(2+2)^2}\right)(-0.05)=-0.0375$

27) $\dfrac{-x}{\sqrt{1-x^2}}dx$

28) $\left(5e^{5x}+5x^4\right)dx$

29) $\dfrac{1}{1+(4x)^2}dx=\dfrac{1}{1+16x^2}dx$

30) $\left(8^x\ln 8+8x^7\right)dx$

31) a) $\Delta f=f(0.1)-f(0)=0.21-0=0.21$

b) $df=(2x+2)dx=2(0+1)(0.1)=0.2$

c) $\left|\Delta f-df\right|=\left|0.21-0.2\right|=0.01$

32) a) $\Delta f=f(1.1)-f(1)=0.231-0=0.231$

b) $df=\left(3x^2-1\right)dx=(3-1)(0.1)=0.2$

c) $\left|\Delta f-df\right|=\left|0.231-0.2\right|=0.031$

33) a) $\Delta f = f(0.55) - f(0.5) = 1.8181818... - 2 = -0.18181818...$

b) $df = \left(\dfrac{-1}{x^2}\right)dx = \left(\dfrac{-1}{0.5^2}\right)(0.05) = -0.2$

c) $\left|\Delta f - df\right| = \left|-0.181818... + 0.2\right| = 0.0181818...$

34) a) $\Delta f = f(1.01) - f(1.) = 1.04060406... - 1 = 0.04060401...$

b) $df = \left(4x^3\right)dx = \left(4(1)^3\right)(0.01) = 0.04$

c) $\left|\Delta f - df\right| = \left|0.04060401... - 0.04\right| = 0.0006040601...$

35) $\Delta V \approx dV = 4\pi r^2 dr \text{ cm}^3 \qquad \Delta V \approx dV = 4\pi(10)^2(0.05) = 20\pi \text{ cm}^3$

36) $\Delta A \approx dA = 8\pi r dr \text{ cm}^2 \qquad \Delta A \approx dA = 8\pi(10)(0.05) = 4\pi \text{ cm}^2$

37) $\Delta V \approx dV = 3x^2 dx \text{ cm}^3 \qquad \Delta V \approx dV = 3(10)^2(0.05) = 15 \text{ cm}^3$

38) $\Delta S \approx dS = 12x\, dx \text{ cm}^2 \qquad \Delta S \approx dS = 12(10)(0.05) = 6 \text{ cm}^2$

39) $\Delta V \approx dV = 2\pi rh\, dr \text{ cm}^3 \qquad \Delta V \approx dV = 2\pi(10)h(0.05) = \pi h \text{ cm}^3$

40) $\Delta S \approx dS = 2\pi r\, dh \text{ cm}^2 \qquad \Delta S \approx dS = 2\pi r(0.05) = 0.1\pi r \text{ cm}^2$

41) $\Delta A \approx dA = 2\pi r\, dr \text{ in}^2 \qquad \Delta A \approx dA = 2\pi(10)(0.1) = 2\pi \text{ in}^2 \approx 6.3 \text{ in}^2$

42) $\Delta V \approx dV = 4\pi r^2\, dr \text{ in}^2 \qquad \Delta V \approx dV = 4\pi(8)^2(0.3) \approx 241.3 \text{ in}^2$

43) $\Delta V \approx dV = 3s^2 ds \text{ cm}^3 \qquad \Delta V \approx dV = 3(15)^2(0.2) = 135 \text{ cm}^3$

44) $A = \dfrac{s^2}{4}\sqrt{3} \quad \Delta A \approx dA = \dfrac{s}{2}\sqrt{3}\, ds \text{ cm}^2 \qquad \Delta A \approx dA = \dfrac{20}{2}\sqrt{3}(0.5) \approx 8.7 \text{ cm}^2$

45) a) $L(x) = 1 + 1(x - 0) = x + 1$

b) $f(0.1) \approx L(0.1) = 1.1$

46) a) $\Delta A \approx dA = 2\pi r\, dr \text{ m} \qquad \Delta A \approx dA = 2\pi(2.00)(0.02) = 0.08\pi \approx 0.251 \text{ m}$

b) $A = \pi r^2 = \pi(2)^2 \approx 12.566 \text{ m} \quad \dfrac{0.251}{12.566} \approx 0.01997 \approx 2\%$

47) $C = \pi d = 10\pi \quad dC = \pi\, dd \quad 2 = \pi\, dd \Rightarrow dd = \dfrac{2}{\pi} \approx 0.637 \text{ in.}$

Cross-section area: $\Delta A \approx dA = 2\pi r\, dr \text{ m} \quad \Delta A \approx dA = 2\pi(5)\left(\dfrac{1}{\pi}\right) = 10 \text{ in}$

48) $V = e^3 = 1000 \text{ cm}^3 \quad \Delta V \approx dV = 3e^2 de = 3(10)^2(0.1) = 30 \quad \frac{30}{1000} = 3\%$

49) $A = s^2 \quad \Delta A \approx dA = 2s\,ds \quad 0.02s^2 = 2s\,ds \Rightarrow ds = 0.01s$. Therefore the side measurement should be within 1% of the actual measurement.

50) a)

$$V = \pi r^2 h \Rightarrow dV = 2\pi rh\,dr \quad r = \frac{d}{2} \Rightarrow dr = \frac{dd}{2}$$

$$\Delta V \approx dV = 2\pi\left(\frac{d}{2}\right)(10)\left(\frac{dd}{2}\right) = 5\pi d\,dd \quad 0.01\left(\pi\left(\frac{d}{2}\right)^2(10)\right) = 5\pi d\,dd \Rightarrow dd = 0.005d$$

The diameter must be measured within 0.5% of its true value for the volume to be within 1% of its true value.

b) $L = \pi dh \Rightarrow dL = \pi h\,dd \quad 0.05(\pi d(10)) = \pi(10)\,dd \Rightarrow dd = 0.05d$

The diameter must be measured within 5% of its true value for the lateral area to be within 5% of its true value.

51) This problem has problems

$$V = \pi r^2 h \Rightarrow dV = 2\pi rh\,dr \quad dV = 2\pi rh(0.001) = 0.002\pi rh$$