AP Calculus Exam Prep Assignment #2 Name ______________________

Multiple Choice

1)

$$V = \pi\int_0^{2\pi}(1-\cos\phi)^2\,d\phi = \pi\int_0^{2\pi}(1-2\cos\phi+\cos^2\phi)\,d\phi = \pi\int_0^{2\pi}\left(1-2\cos\phi+\frac{1+\cos 2\phi}{2}\right)d\phi$$

D)

$$= \pi\left[\frac{3}{2}\phi + 2\sin\phi + \frac{1}{4}\sin 2\phi\right]_0^{2\pi} = \pi\left[(3\pi+0+0)-(0+0+0)\right] = 3\pi^2$$

2) $A = \int_0^{2\pi}(1-\cos x)\,dx = \left[x-\sin x\right]_0^{2\pi} = (2\pi-0)-(0-0) = 2\pi$ **E)**

3)

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\frac{-t^2+3}{3}}{e^t} = \frac{-t^2+3}{3e^t} \qquad x-\text{int} \Rightarrow y=0 \Rightarrow t\left(-\frac{t^2}{9}+1\right)=0 \Rightarrow t = 0, \pm 3$$

A)

$$\frac{dy}{dx}(-3) = \frac{-9+3}{3e^{-3}} = -2e^3$$

4)

$$\frac{dy}{dt} = 4\cos t + 12\cos 12t \qquad \frac{dy}{dt}(1) = 4\cos 1 + 12\cos 12 \approx 12.287$$

E)

5) Find $\frac{d^2y}{dx^2}$ if $x = 2\cos\phi$ and $y = \sin\phi$.

$$\frac{dy}{dx} = \frac{\cos\phi}{-2\sin\phi} = -\frac{1}{2}\cot\phi \qquad \frac{d^2y}{dx^2} = \frac{\frac{1}{2}\csc^2\phi}{-2\sin\phi} = -\frac{1}{4}\csc^3\phi$$

A)

6) The length of $r = 3\csc\phi$ from $\phi = \frac{\pi}{4}$ to $\phi = \frac{3\pi}{4}$ is:

$$x = r\cos\theta = 3\csc\theta\cos\theta = 3\cot\theta \Rightarrow \frac{dx}{d\theta} = -3\csc^2\theta$$

$$y = r\sin\theta = 3\csc\theta\sin\theta = 3 \Rightarrow \frac{dy}{d\theta} = 0$$

A)

$$L = \int_{\pi/4}^{3\pi/4}\sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta = \int_{\pi/4}^{3\pi/4}\sqrt{9\csc^4\theta+0}\,d\theta = \int_{\pi/4}^{3\pi/4}3\csc^2\theta\,d\theta = \left[-3\cot\theta\right]_{\pi/4}^{3\pi/4} = 3-(-3) = 6$$

7) Find the slope of the curve $r=\cos 2\phi$ at $\phi=\dfrac{\pi}{6}$

$$x=\cos 2\phi(\cos\phi) \quad y=\cos 2\phi(\sin\phi) \quad \frac{dy}{dx}=\frac{\cos 2\phi(\cos\phi)-2\sin 2\phi(\sin\phi)}{-\cos 2\phi(\sin\phi)-2\sin 2\phi(\cos\phi)}$$

A)

$$\frac{dy}{dx}\left(\frac{\pi}{6}\right)=\frac{\frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)-2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{-\frac{1}{2}\left(\frac{1}{2}\right)-2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}=-\frac{\sqrt{3}}{4}\cdot\frac{4}{-7}=\frac{\sqrt{3}}{7}$$

8) Find the first quadrant area inside the rose, $r=3\sin 2\phi$, but outside the circle, $r=2$

$$A=\frac{1}{2}\int_{\alpha}^{\beta}r^2d\theta \quad 3\sin 2\phi=2\Rightarrow\sin 2\phi=\frac{2}{3}\Rightarrow\phi\approx 0.365,1.206$$

D)

$$A=\frac{1}{2}\int_{0.365}^{1.206}\left[(3\sin 2\phi)^2-(2)^2\right]d\phi\approx 1.328$$

9) The common area inside $r=2\sin\phi$ and $r=2\cos\phi$ is:

$$A=\frac{1}{2}\int_{\alpha}^{\beta}r^2d\theta \quad A=2\left(\frac{1}{2}\int_{\pi/4}^{\pi/2}(2\cos\phi)^2\,d\phi\right)=\int_{\pi/4}^{\pi/2}4\cos^2\phi\,d\phi=4\int_{\pi/4}^{\pi/2}\frac{1+\cos 2\phi}{2}\,d\phi=2\int_{\pi/4}^{\pi/2}(1+\cos 2\phi)\,d\phi$$

B)

$$2\left[\phi+\frac{1}{2}\sin 2\phi\right]_{\pi/4}^{\pi/2}=2\left[\left(\frac{\pi}{2}+0\right)-\left(\frac{\pi}{4}+\frac{1}{2}\right)\right]=\frac{\pi}{2}-1$$

10) The area enclosed by the rose, $r=\cos 2\phi$ is:

$$A=\frac{1}{2}\int_{\alpha}^{\beta}r^2d\theta \quad A=\frac{1}{2}\int_{0}^{2\pi}(\cos 2\phi)^2\,d\phi=\frac{1}{2}\int_{0}^{2\pi}\frac{1+\cos 4\phi}{2}\,d\phi=\frac{1}{4}\int_{0}^{2\pi}d\phi+\frac{1}{4}\int_{0}^{2\pi}\cos 4\phi\,d\phi$$

B)

$$\frac{\pi}{2}+\frac{1}{16}\left[\sin 4\phi\right]_0^{2\pi}=\frac{\pi}{2}$$