AP Calculus Exam Prep Assignment #11 KEY

1) The equation of the tangent to the curve $2x^2 - y^4 = 1$ at (-1,1) is:

$4x - 4y^3\frac{dy}{dx} = 0 \quad \frac{dy}{dx} = \frac{x}{y^3} \quad$ At $(-1,1)\ \frac{dy}{dx} = \frac{1}{-1} = -1 \quad y - 1 = -1(x+1) \Rightarrow = -x$ **A)**

2) The tangent to $y = xe^{-x}$ is horizontal when $x =$ _____.

$y' = e^{-x} - xe^{-x} = e^{-x}(1-x) \quad y' = 0 \Rightarrow x = 1$ **B)**

3) An equation for a tangent to $y = \arcsin\left(\frac{x}{2}\right)$ at the origin is:

$y' = \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}\left(\frac{1}{2}\right) = \frac{1}{2\sqrt{1-\frac{x^2}{4}}} \quad y = \frac{1}{2}x \Rightarrow x - 2y = 0$ **A)**

4) The approximate value of $y = \sqrt{4+\sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is:

$y' = \frac{1}{2}(4+\sin x)^{-1/2}(\cos x) \quad y = \sqrt{4+0} = 2 \quad y - 2 = \frac{1}{4}x \Rightarrow y = \frac{1}{4}x + 2 \quad y = \frac{1}{4}(0.12) + 2 = 2.03$ **B)**

5) If $f(x) = \ln x$, then the average rate of change of f on the interval [2,5] is:

$\frac{\ln 5 - \ln 2}{5-2} = \frac{1}{3}\ln\frac{5}{2} \approx 0.305$ **C)**

6) If $f(x) = \sqrt[5]{x^3 - 2x}$, then $\left(\sqrt{3}\right) =$ **D) 1.116**

7) [GC] The slope of the line containing the point (4,0) and tangent to the graph of $y = e^{-x}$ is:

$y' = -e^{-x}$ Let $\left(x, e^{-x}\right)$ be the point of tangency

$-e^{-x} = \frac{e^{-x} - 0}{x - 4} \Rightarrow x - 4 = -1 \Rightarrow x = 3,$ so the slope is $-e^{-3} \approx -0.050$ **E)**

8) If c is the number defined by Rolle's Theorem, then for $f(x) = 2x^3 - 6x$ on the interval $0 \le x \le \sqrt{3}$, $c =$:

$f'(x) = 6x^2 - 6 \quad f'(x) = 0 \Rightarrow 6x(x-1) = 0 \Rightarrow x = 0,1$ Since 0 is an endpoint of the interval, the number is 1.

A)

9) If $f'(x)$ exists on [a,b], it follows that:

E) the Mean Value Theorem applies

10) The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between (0,0) and (4,2). What are the coordinates of this point?

$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{2}{4} \quad f'(x) = \frac{1}{2\sqrt{x}} \quad \frac{1}{2\sqrt{c}} = \frac{1}{2} \Rightarrow \sqrt{c} = 1 \Rightarrow c = 1 \quad (1,1)$ **B)**

11) The function $f(x) = x^{2/3}$ on [-8,8] does not satisfy the conditions of the Mean Value Theorem because:

E) $f'(0)$ **does not exist**