What you'll learn about

- Continuity at a Point
- Continuous Functions
- Algebraic Combinations
- Composites
- Intermediate Value Theorem for Continuous Functions

…and why

Continuous functions are used to describe how a body moves through space and how the speed of a chemical reaction changes with time.

EQ:

What is a continuous function, and how can we use it in applications?

Continuity at a Point

Any function $f(x)$ whose graph can be sketched in one continuous motion without lifting the pencil is an example of a continuous function.

Continuity at a Point

Interior Point: A function $y = f(x)$ is continuous at an interior point c of its domain if $\lim_{x \to c} f(x) = f(c)$.

Endpoint: A function $y = f(x)$ is continuous at a left endpoint a or is continuous at a right endpoint b of its domain if

$$\lim_{x \to a^+} f(x) = f(a) \quad \text{or} \quad \lim_{x \to b^-} f(x) = f(b) \quad \text{respectively.}$$

p. 79 EQ: What is a continuous function, and how is it used?

Continuity at a Point

If a function f is **not continuous at a point c**, we say that f is **discontinuous at c** and c is a point of discontinuity of f.

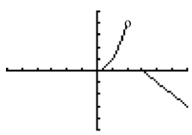
Note that c need not be in the domain of f.

Slide 2- 7

EQ: What is a continuous function, and how is it used?

Example **Continuity at a Point**

Find the points at which the given function is continuous and the points at which it is discontinuous.



Points at which f is continuous:

At $x = 0$ $\quad \lim_{x\to 0^+} f(x) = f(0)$

At $x = 6$ $\quad \lim_{x\to 6^-} f(x) = f(6)$

At $0 < c < 6$, but not $2 \le c < 3$ $\quad \lim_{x\to c} f(x) = f(c)$

Points at which f is discontinuous:

At $x = 2$ $\quad \lim_{x\to 2} f(x)$ does not exist.

At $c < 0,\ 2 < c < 3,\ c > 6$ $\quad$ These points are not in the domain of f.

Slide 2- 8

p. 80 EQ: What is a continuous function, and how is it used?

Continuity at a Point

The typical discontinuity types are:

1) Removable

2) Jump

3) Infinite

4) Oscillating

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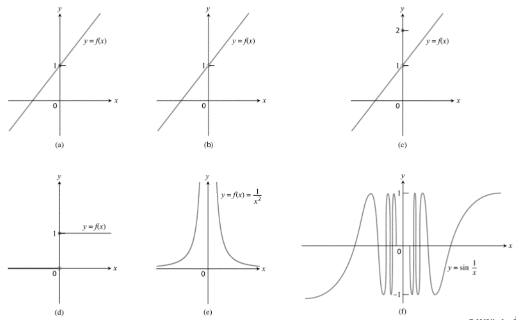
p. 80 EQ: What is a continuous function, and how is it used?

Continuity at a Point



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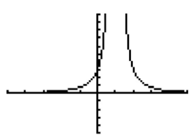
EQ: What is a continuous function, and how is it used?

Example **Continuity at a Point**

Find and identify the points of discontinuity of $y = \dfrac{3}{(x-1)^2}$



[-5,5] by [-5,10]

There is an infinite discontinuity at $x = 1$.

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p. 81 EQ: What is a continuous function, and how is it used?

Continuous Functions

A function is **continuous on an interval** if and only if it is continuous at every point of the interval. A **continuous function** is one that is continuous at every point of its domain. A continuous function need not be continuous on every interval.

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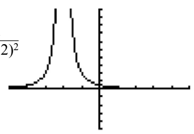
EQ: What is a continuous function, and how is it used?

Continuous Functions

The given function is a continuous function because it is continuous at every point of its domain. It does have a point of discontinuity at $x = -2$ because the function is not defined at that point.

$$y = \frac{2}{(x+2)^2}$$



[-5,5] by [-5,10]

Important distinction: the function is a *continuous function*, but the function is NOT continuous on the interval [-3,1]!!!!

A continuous function can have a discontinuity!

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p. 82 EQ: What is a continuous function, and how is it used?

Properties of Continuous Functions: <u>Theorem 6</u>

If the functions f and g are continuous at $x = c$, then the following combinations are continuous at $x = c$.

1. Sums $f + g$
2. Differences: $f - g$
3. Products: $f \bullet g$
4. Constant multiples $k \bullet g$, for any number k
5. Quotients $\frac{f}{g}$, provided $g(c) \neq 0$

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p. 82 EQ: What is a continuous function, and how is it used?

Composite of Continuous Functions: <u>Theorem 7</u>

If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

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p. 83 EQ: What is a continuous function, and how is it used?

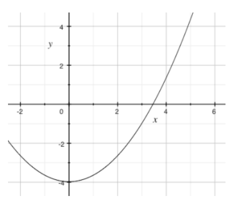
Intermediate Value Theorem for Continuous Functions: Theorem 8

A function $y = f(x)$ that is continuous on a closed interval $[a,b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = f(c)$ for some c in $[a,b]$.

EQ: What is a continuous function, and how is it used?

Intermediate Value Theorem for Continuous Functions

Example: Graph $f(x) = \frac{1}{3}x^2 - 4$ on [-1,5]



$f(-1) = \frac{-11}{3}$

$f(5) = \frac{13}{3}$

By the IVT, $f(x)$ must contain ALL values on $\left[\frac{-11}{3}, \frac{13}{3}\right]$

p. 83 EQ: What is a continuous function, and how is it used?

Intermediate Value Theorem for Continuous Functions

The Intermediate Value Theorem for Continuous Functions is the reason why the graph of a function continuous on an interval cannot have any breaks. The graph will be **connected**, a single, unbroken curve. It will not have jumps or separate branches.